Two-stage Spectrum Sharing with Combinatorial Auction and Stackelberg Game in Recall-based Cognitive Radio Networks

Changyan Yi and Jun Cai, Senior Member, IEEE

Abstract—The dynamic spectrum access (DSA) among multiple heterogeneous primary spectrum owners (POs) and secondary users (SUs) in recall-based cognitive radio networks is investigated in this paper. In our framework, SUs demand a different amount of spectrum for their transmissions. Each PO provides a portion of radio resources for leasing and also offers its own primary users (PUs) a certain degree of quality of service (QoS). Furthermore, POs are allowed to have different spectrum trading areas and as well as heterogeneous activities between POs’ users. We propose a Two-stage resource allocation scheme with combinatorial Auction and Stackelberg Game in spectrum Sharing (TAGS) to deal with the allocation problem in such a complicated system. In the first stage, a spectrum allocation is decided by running a geographically restricted combinatorial auction without the consideration of spectrum recall. In the second stage, a Stackelberg game is formulated for all users to determine their best strategies with respect to the potential spectrum recall. Both theoretical and simulation results prove that TAGS provides a feasible solution for the problem and ensures the desired economic properties for all individuals.

Index Terms—Cognitive radio networks, auction, dynamic spectrum access, quality of service, Stackelberg game.

I. INTRODUCTION

W ith the explosive growth of demand for radio resources from newly developed wireless equipment and applications, the existing spectrum regulatory policy leads to a severe spectrum scarcity because of its static assignment pattern [1]. Therefore, there is an immense need to dynamically exploit the under-utilized licensed spectrum in order to enhance the spectrum utilization. Cognitive radio (CR) has been proposed as a solution to provide secondary users (SUs) with opportunistic access to the licensed spectrum owned by primary users (PUs) [2], [3]. Unlike the non-profit spectrum sharing on optimizations of cognitive radio networks [4]–[6], spectrum marketing [7] has been attracting more and more attention in recent years, because PUs can obtain potential economic profit by temporarily leasing their unused spectrum.

Spectrum auction has been widely discussed in literature [8], [9] and is one of the most effective spectrum marketing methods. For example, Feng et al. in [10] studied an auction mechanism in a situation where spectrum shows distinctive characteristics in both spatial and frequency domains. In [11], Wang et al. proposed the idea of auction coverage, where each licence holder could partition its license area and decide whether to proceed with spectrum auctions in specific areas. Both of these works assumed homogeneity in the amount of spectrum for sale or for demand, so that the assignment problem could be simplified to a matching problem between sellers and buyers. Some other works in this area include a cooperation-based dynamic spectrum leasing mechanism via multi-winner auction for multiple bands [12], and a framework for multi-channel spectrum auction where both PUs and SUs are able to trade multiple items [13]. A common assumption in all aforementioned works is that the auctioned spectrum is occupied exclusively by the winning SUs. Such an assumption imposes a dilemma for license holders requiring them to either auction their unused spectrum and get revenue at the risk of a sudden increase of spectrum demand from PUs, or reserve spectrum uneconomically. To address this issue, Wu et al. in [14] introduced a new framework of recall-based dynamic spectrum auction, where the spectrum seller could recall the auctioned spectrum from winners whenever doing so could enhance its utility. However, this framework may be too primitive for practical applications as it assumes only one seller and a homogeneous spectrum demand on the part of all buyers.

Besides spectrum auction, game theory has also been widely applied for studying spectrum marketing in CR networks. Among the many game models, the Stackelberg game is one that can be used to study the interaction between users who have different levels of control or information in spectrum marketing [15], [16]. For instance, in [17], the PU and SUs were modeled as leader and followers respectively, and the spectrum management of PU was studied by setting up a pricing function. The authors in [18] discussed the pricing policy of service providers in one-PU multiple-SU CR networks and showed that to maximize the revenue, users with better channel conditions and more willingness to pay for the provided service should be charged higher prices. In [19], a multiple-PU market is considered and a three-stage Stackelberg game is formulated where each SU selects one of PUs’ channels to maximize its throughput.

In this paper, we readdress the spectrum sharing issue in CR networks by integrating advanced features such as spectrum recall and local trading markets. A CR network with multiple heterogeneous primary spectrum owners (POs) and SUs is considered. Each PO has a different amount of spectrum to lease in different specific areas, and has a specific PUs’ activity. Each SU has heterogeneous requirements in terms
of spectrum demands and attitudes towards POs’ potential recall. Obviously, under this scenario, spectrum sharing needs to jointly consider both spectrum allocation and individual strategies. However, solving such a joint optimization problem is challenging due to the facts that i) PUs’ activities are random and heterogeneous among all POs, and ii) before the spectrum allocation has been done, the quantity of spectrum recalled from each SU is impossible to know. In order to deal with the high computational complexity involved in solving such a joint optimization problem, we decompose it into two separate stages and propose a new method called Two-stage resource allocation scheme with combinatorial Auction and Stackelberg Game in spectrum Sharing (TAGS). In stage I, an optimal spectrum allocation is derived by formulating a combinatorial spectrum auction without considering spectrum recall. Based on the winner determination in the auction, each PO decides a maximum quantity of spectrum recall in stage II and each winning SU claims a payment reduction so as to offset the risk of utility degradation. We formulate such a decision making process as a Stackelberg pricing game and figure out the best strategies for both POs and SUs. Theoretical and simulation results demonstrate that the proposed scheme is rational and economically feasible.

To our best knowledge, our work is the first spectrum sharing mechanism which considers multiple heterogeneous spectrum sellers and buyers in recall-based cognitive radio networks. The main contributions of this paper are as follows:

- A complicated recall-based cognitive radio network is studied, which consists of multiple heterogeneous spectrum sellers and buyers.
- A new spectrum sharing mechanism, called TAGS, is proposed, which decomposes the spectrum sharing process into two stages, i.e., a combinatorial spectrum auction and a recall-based pricing game.
- Winner determination problem (WDP) of the geographically restricted combinatorial spectrum auction is analyzed and an approximate auction mechanism is proposed.
- The recall-based strategy decision process is formulated as a Stackelberg game, and the existence and uniqueness of Nash equilibrium (NE) are proved.
- Both theoretical and numerical analyses are provided to show the economic feasibility of the proposed scheme.

The rest of the paper is organized as follows: Section II describes the system model and summarizes all important notations used in this paper. Section III presents the first stage of TAGS, i.e., the combinatorial spectrum auction, and proposes an approximate auction mechanism. A recall-based pricing game is formulated in Section IV to study the potential strategy decision process in the second stage. Section V shows the analyses of desired economic properties and presents a detailed time-line of our proposed TAGS. Simulation results are illustrated in Section VI. Finally, we give a brief conclusion in Section VII.

II. SYSTEM MODEL

Consider a CR network consisting of $m$ POs and $n$ SUs as shown in Fig. 1. Each PO $i$ owns bandwidth $W_i$ to serve its own subscribed PUs. We assume that PUs with the same PO $i$ are homogeneous in terms of spectrum demand $s^i$ and individual utility $u^i$ (e.g., $u^i$ could be set as the valuation of achievable rate by receiving $s^i$). However, PUs from different POs may be heterogeneous. If the remaining spectrum of PO $i$ is less than $s^i$, newly arrived PUs have to wait in the queue and will be served later based on the first-come-first served (FCFS) rule. Without loss of generality, PUs arrive at each PO $i$ following a Poisson process with an average arrival rate of $\lambda_i$. Furthermore, assume that the spectrum occupancy time of PUs in PO $i$ is independent and identically exponentially distributed with service rate $\mu_i$. Then, the service of PUs in PO $i$ could be regarded as a $M/M/c$ queueing system with $c = \lceil W_i/s^i \rceil$.

Each PO could predefine a specific geographic region for spectrum marketing and lease a certain quantity of spectrum to SU within this area, while at the same time guaranteeing its PUs’ quality of service (QoS). We define the mean waiting time in the queue as the measurement of QoS for PUs. Specifically, if the mean waiting time of PUs in PO $i$, i.e., $M_W$ is longer than a certain requirement $\alpha_i$, PO $i$ has to be punished for the QoS degradation.

Assume that all the POs are synchronized with same time frames and spectrum sharing is carried out frame by frame. At the beginning of each frame, each PO determines the quantity of spectrum for leasing based on its own PUs’ current spectrum usage. However, due to the random activities of PUs, POs may have insufficient spectrum to serve a sudden increasing demand from their own PUs if their unused spectrum has already been auctioned off. Since PUs are granted with higher spectrum access priority, we enable spectrum recall for POs, i.e., each PO could recall some auctioned spectrum from the winning SUs to satisfy its own PUs’ demands if necessary. Note that POs are not necessary to recall spectrum for all newly arrived users. In fact, each PO can tolerate suffering from a degradation on PUs’ QoS if its overall utility can be improved. Moreover, we assume that recalled spectrum would not be returned to SUs until the next time frame. Certainly, winning SUs will get corresponding refunds if their spectrum were recalled by POs.

All SUs are assumed to be located within their interference ranges so that spectrum spatial reuse is not considered in this work. Similar assumptions have been used in small SUs’
networks [20], [21]. We will consider potential spectrum reuse in our future works. Furthermore, since it is difficult in employing discontinuous spectrum bands from different operators (POs) for a radio device with limited physical layer capability [20], [22], we assume that each SU can only access the spectrum from a single PO.

There is a small period used for spectrum management at the beginning of each frame. Since each PO predefines its specific region for spectrum leasing and SUs are randomly scattered in the entire area, it is difficult to find an optimal spectrum allocation in a distributed manner. Thus, a central entity, called spectrum broker, is introduced in the network. However, even with the central broker, it is still hard, if possible, to jointly determine the optimal spectrum allocation and best spectrum recall strategies because 1) the optimal amount of spectrum recall from each PO relies on a pre-existing optimal winner assignment; 2) the optimal winner assignment should be based on the optimal bids collected from SUs; and 3) the optimal bids are in turn determined regarding to the potential spectrum recall. In order to tackle the complexity of this issue, we propose a two-stage solution, called TAGS as illustrated in Fig. 2. In the first stage, each PO reports to the spectrum broker the quantity of spectrum for leasing and its specific spectrum trading region. Note that since the recall information is unknown in the first stage, the amount of spectrum recalled by POs cannot be considered as strategies in the auction. At the same time, each SU sends out its private information, including its spectrum demand, its bidding price, and location. The broker collects all these sealed-bid information and determines an optimal allocation which leads to a social optimality without considering the spectrum recall. In addition, the broker calculates the payments and payoffs for SUs and POs respectively. In the second stage, each PO informs its own winning SUs a maximum quantity of spectrum which may be recalled, along with its spectrum recall scheme. Each SU then determines a reduction on its payment so that its utility could be maximized with such strategy. Finally, POs in turn derive the spectrum recall ratio distributed on each winner.

For convenience, Table I lists some important notations used in this paper.

![Fig. 2. An illustration of all actions in TAGS](image)

### III. FIRST STAGE: COMBINATORIAL SPECTRUM AUCTION

In this section, a centralized combinatorial spectrum auction is introduced that will be applied in the first stage. We first formulate the winner determination problem (WDP) by binary integer programming (BIP). Due to the computational intractability of BIP, we propose a new strategy-proof combinatorial auction mechanism which runs in a polynomial time.

### A. Winner Determination Problem (WDP)

The defined combinatorial spectrum auction consists of \( m \) sellers (POs), each with heterogeneous amount of goods (spectrum) to sell, and \( n \) bidders (SUs) with different demands. Define the set of POs as \( M \) with \(|M| = m\). Each PO \( i \) determines the quantity of spectrum for leasing, called \( C_i \in \mathbb{R}, i \in M \), based on its own PUs’ activities. For simplicity, we assume that the spectrum trading area of each PO \( i \) is a circle in an Euclidean plane with the location of this PO as the center and the radius of \( R_i \).

Similarly, we define \( N \) as the set of SUs with \(|N| = n\). Each SU \( j \) has a specific spectrum demand, \( d_j \in \mathbb{R}, j = 1, 2, \ldots, n \), and a private valuation for its demand, \( \nu_j, j = 1, 2, \ldots, n \). Without loss of generality, let \( \nu_j \) equal the monetary value of Shannon capacity that SU \( j \) could obtain over \( d_j \) spectrum bandwidth as

\[
v_j = \sigma_j d_j \log_2(1 + \eta_j)
\]

where \( \sigma_j \) and \( \eta_j \) are the monetary weight index and the signal-to-noise ratio (SNR), respectively. Both \( \sigma_j \) and \( \eta_j \) are supposed to be constant for each SU \( j \) [23].

At the beginning of the auction, POs report their auction information \( \{A_1, \ldots, A_m\} \), to the spectrum broker. Here, each \( A_i \) is a 3-tuple \((C_i, O_i, R_i)\), where

- \( C_i \) is the spectrum bandwidth provided by PO \( i \) for leasing.
- \( O_i \) represents the location of seller \( i \), which is assumed to be a coordinate in Euclidean plane, i.e., \( O_i = (o_i^x, o_i^y) \).
- \( R_i \) denotes the radius of PO \( i \)'s spectrum trading region.

Meanwhile, SUs send their bids to the spectrum broker, denoted as \( B_j, j = 1, 2, \ldots, n \). Each bid \( B_j \) is also specified as a 3-tuple \((d_j, p_j, l_j)\), where

### TABLE I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>( m )</td>
<td>number of primary spectrum owners (POs)</td>
</tr>
<tr>
<td>( n )</td>
<td>number of secondary users (SUs)</td>
</tr>
<tr>
<td>( W_i )</td>
<td>total spectrum bandwidth of PO ( i )</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>mean waiting time requirement of PO ( i )</td>
</tr>
<tr>
<td>( C_i )</td>
<td>amount of auctioned spectrum offered by PO ( i )</td>
</tr>
<tr>
<td>( e_j )</td>
<td>status indicator of SU ( j )</td>
</tr>
<tr>
<td>( q_j )</td>
<td>payment of SU ( j ) determined in the first stage</td>
</tr>
<tr>
<td>( I_i )</td>
<td>payoff that each PO obtain after the auction</td>
</tr>
<tr>
<td>( \Theta_i )</td>
<td>the set of winning SUs allocated to PO ( i )</td>
</tr>
<tr>
<td>( \omega_i )</td>
<td>spectrum recall ratio determined by PO ( i )</td>
</tr>
<tr>
<td>( RC_i )</td>
<td>maximum amount recalled by PO ( i )</td>
</tr>
<tr>
<td>( \beta_j )</td>
<td>payment reduction parameter of winning SU ( j )</td>
</tr>
<tr>
<td>( r_j )</td>
<td>quantity of spectrum recall on SU ( j )</td>
</tr>
</tbody>
</table>
where \( x \) is the spectrum demand of bidder \( j \).
- \( p_j \) indicates the amount that the bidder is willing to pay for \( d_j \). Note that, for truthful auction, the bidding price equals the true valuation, i.e., \( p_j = v_j \). Truthfulness of our auction is proved later in Section III-B3.
- \( l_j \) represents the location of bidder \( j \), which is assumed to be a coordinate in Euclidean plane, i.e., \( l_j = (l_{jx}, l_{jy}) \).

After receiving all these sealed information, the spectrum broker first identifies the locations of all bidders and sellers, and then groups the bidders into \( m \) sets according to the spectrum trading area of each seller. Note that these \( m \) sets may be overlapped, i.e., each SU could be located in multiple POs’ trading areas. Let \( V_i \) denote the set of bidders who locate in the auction coverage of PO \( i \). Obviously, all bidders in set \( V_i \) should satisfy the following condition:

\[
\sqrt{(l_{jx} - o_i^x)^2 + (l_{jy} - o_i^y)^2} \leq R_i, \quad \forall j \in V_i. \tag{2}
\]

After grouping the bidders, the spectrum broker formulates an optimization problem to determine the winners in order to maximize the social welfare, i.e., the total bidding price from all winning SUs. The formulated optimization problem is

\[
\max_{\{x_{ij}, \forall i \in M, \forall j \in N\}} \sum_{i=1}^{m} \sum_{j=1}^{n} p_j x_{ij} \tag{3}
\]

s.t. \[
\sum_{j \in V_i} d_j x_{ij} \leq C_i, \quad \forall i \in M, \tag{4}
\]

\[
\sum_{i=1}^{m} x_{ij} \leq 1, \quad \forall j \in N, \tag{5}
\]

\[
x_{ij} \in \{0, 1\}, \quad \forall i \in M, \forall j \in N. \tag{6}
\]

where \( x_{ij} = 1 \) if SU \( j \) is allocated to PO \( i \), and \( x_{ij} = 0 \), otherwise.

The first constraint means that for winning SUs allocated to PO \( i \), their total demands should be less than or equal to the quantity of spectrum PO \( i \) could offer. The second constraint limits each SU to access spectrum from no more than one PO. The third constraint assumes that all bidders are single-minded so that there are only two outcomes for each SU, i.e., win or lose. If the optimal winners assignment can be obtained, then the well-known Vickrey-Clarke-Groves (VCG) mechanism [24] can be applied to calculate the payments in order to ensure both strategy-proofness and efficiency of the auction. However, the above WDP is obviously a BIP which can be proved as NP-hard by reducing it to a weighted independent set problem [25]. In the next subsections, we will propose an alternative mechanism with an approximate winner determination algorithm and a tailored payment scheme.

### B. Auction Mechanism

1) **Approximate Algorithm for WDP**: Inspired by the approximation algorithm for multiple knapsack problem (MKP) in [26], we introduce the following algorithm to solve the formulated WDP in a polynomial time. Note that, different from the general MKP, where items could be allocated to any knapsack without considering geographic restriction, our proposed algorithm takes both the POs’ auction region and SUs’ locations into account.

After receiving all the bids, according to the idea of LOS algorithm [27], the spectrum broker first sorts the SUs based on a decreasing order of \( \frac{p_j}{d_j} \), \( j = 1, \ldots, n \), and sorts POs based on an increasing order of the amount of auctioned spectrum, \( C_i, i = 1, \ldots, m \), i.e.,

\[
\frac{p_1}{\sqrt{d_1}} \geq \frac{p_2}{\sqrt{d_2}} \geq \cdots \geq \frac{p_j}{\sqrt{d_j}} \geq \cdots \geq \frac{p_n}{\sqrt{d_n}} \tag{7}
\]

\[
C_1 \leq C_2 \leq \cdots \leq C_i \leq \cdots \leq C_m \tag{8}
\]

Notice that, although it is straightforward to order the SUs with their unit bidding price, i.e., \( \frac{p_j}{d_j}, \forall j \in N \), such order may undervalue bids with large demands. In fact, [27] has proved that ordering bidders with \( \frac{p_j}{d_j} \) would provide a better approximation ratio to optimality. In addition, in (7) and (8), all the indices have been rearranged and the following searching procedure will follow this order.

We first derive an initial feasible solution by using the algorithm shown in Algorithm 1. Define \( z \) as the overall bidding price of the auction, \( C_i \), as the remaining capacity (in terms of the spectrum bandwidth) of knapsack (PO) \( i \) and \( e_j \) as the status indicator of SU \( j \), where

\[
e_j = \begin{cases} 
0, & \text{if SU } j \text{ is currently unallocated} \\
1, & \text{index of the PO it is allocated to, otherwise}
\end{cases}
\]

The proposed algorithm considers the POs one by one. For any PO \( i \), the algorithm assigns the spectrum to the unallocated SUs which are covered in the auction region of PO \( i \) until the remaining capacity is smaller than the request from any unallocated SUs. For each feasible allocation, the algorithm updates the following parameters as \( e_j = i \) and \( z = z + p_j \).

Subsequently, we will improve the derived initial solution based on the idea of local exchanges [26]. The improvement

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**Algorithm 1 Initial Solution for WDP**

1: input: \( n, p_j, d_j, l_j, m, C_i, O_i, R_i \);
2: output: \( z, e_j \);
3: \( z = 0; \)
4: for \( j = 1 \) to \( n \) do
5: \( e_j = 0; \)
6: end for
7: for \( i = 1 \) to \( m \) do
8: \( C_i = C_i, V_i = \emptyset; \)
9: for \( j = 1 \) to \( n \) do
10: if \( \sqrt{(l_{jx} - o_i^x)^2 + (l_{jy} - o_i^y)^2} \leq R_i \) then
11: \( V_i = V_i \cup \{j\}; \)
12: end if
13: for each \( j \in V_i \) do
14: if \( e_j = 0 \) and \( d_j \leq C_i \) then
15: \( e_j = i, C_i = C_i - d_j, z = z + p_j; \)
16: end if
17: end for
18: end for
19: end for
consists of three processes, i.e., rearrangement, interchange, and replacement [28].

- **Rearrangement**
  Consider all SUs with \( e_j > 0 \) according to the increasing order of \( \frac{p_j}{\sqrt{d_j}} \). We rearrange these SUs one by one to its next potential trading PO with sufficient remaining capacity in a cyclic manner, i.e., in the order of \( \{e_j + 1, e_j + 2, \ldots, m, 1, 2, \ldots, e_j - 1\} \). Note that, with this rearrangement, the SUs with less demand may be assigned to the PO with smaller residual capacities so that more capacity in the current PO may be available to unallocated SUs.

- **Interchange**
  The interchange process considers all pairs of allocated SUs and, if possible, interchanges their PO assignment whenever doing so allows insertion of a new SU to one of the knapsacks (POs). Through this algorithm, social welfare (the value of \( s \)) can be enhanced since the number of winning SUs is increased.

- **Replacement**
  This process aims to replace any already allocated SU by one or more unallocated SUs which are also covered in the trading area of the same PO, so that the total profit is increased.

Based on the analysis in [26], it is not difficult to prove that no step of these algorithms needs more than \( O(n^2) \) time. Thus, an approximately optimal solution of the WDP could be achieved by sequentially executing the algorithms presented above in a polynomial time.

2) **Payment Design**: Since the VCG payment rule is incompatible with approximate WDP algorithm in a quite general sense [29], we adopt the idea of LOS pricing scheme [27] for determining charging prices that are “Vickrey-like”. Specifically, the payment of each winning SU \( j \) should be a function of the highest-value bid that \( j \)'s bid blocks.

**Definition 3.1 (blocks)**: Suppose bid \( B_j \) was granted by the WDP algorithm while bids in set \( B_j^- \) were denied. The bid \( B_j \) blocks \( B_j^- \) if, after removing the bid \( B_j \) from the auction, all bids in \( B_j^- \) would be granted.

Based on this definition, we can calculate the payment of each SU \( j \) by distinguishing two cases:

- If \( SU_j \) loses or it wins but blocks no other bid (i.e., \( B_j^- = \emptyset \)), then its payment is 0.
- If \( SU_j \) is granted its demand \( d_j \) and \( B_j^- \neq \emptyset \), the payment \( q_j \) of SU \( j \) is set as

  \[
  q_j = \sqrt{d_j} \times \max_{k \in B_j^-} \left( \frac{p_k}{\sqrt{d_k}} \right) \tag{9}
  \]

  After deciding the charges from all the winning SUs, the spectrum broker is responsible to determine the payoffs to each PO based on the number of SUs allocated to it. The income of PO \( i \), \( I_i \), can be easily derived as

  \[
  I_i = \sum_{j \in V_i} x_{ij} q_j = \sum_{j=1}^{\bar{n}} x_{ij} q_j \tag{10}
  \]

  Note that both \( q_j, j \in N \) and \( I_i, i \in M \) are considered as the contract made by the first-stage spectrum allocation. All the users would follow this contract along with its corresponding spectrum allocation and bring it into the next stage.

3) **Auction Properties**: In this subsection, we prove that the first-stage auction is individually rational and incentively compatible.

**Lemma 3.1**: The auction mechanism in the first stage provides individual rationality for all truthful buyers (i.e. \( p_j = v_j \)).

**Proof**: The utility of SU \( j \) is zero if it loses the auction. Otherwise, the utility of winning SU \( j \) can be calculated as

\[
U_j = v_j - q_j = p_j - \sqrt{d_j} \times \max_{k \in B_j^-} \left( \frac{p_k}{\sqrt{d_k}} \right) \tag{11}
\]

The above inequality holds since SU \( j \) is a winner and thus \( \frac{p_j}{\sqrt{d_j}} \geq \max_{k \in B_j^-} \left( \frac{p_k}{\sqrt{d_k}} \right) \) according to Definition 3.1. Hence, the payment scheme in first stage ensures non-negative utilities.

**Lemma 3.2**: The auction mechanism in the first stage is incentive compatible which means that no buyer could obtain higher utility by bidding untruthfully.

**Proof**: We consider two different cases to prove this lemma:

Case I: SU \( j \) wins the auction and gets utility \( U_j > 0 \) when bidding truthfully. If SU \( j \) bids untruthfully (\( p_j' \neq v_j \)), there could be two possible outcomes, i.e., i) if SU \( j \) is a winner, the auction and \( U_j = 0 \); or ii) SU \( j \) still wins the auction and its utility becomes

\[
\hat{U}_j = v_j - q_j' = v_j - \sqrt{d_j} \times \max_{k \in B_j^-} \left( \frac{p_k}{\sqrt{d_k}} \right) \tag{12}
\]

Obviously, we have \( \hat{U}_j = U_j \).

Case II: SU \( j \) loses the auction when bidding truthfully and get utility \( U_j = 0 \). Its utility may be changed only if SU \( j \) wins with an untruthful bidding. Let \( p_j \) and \( p_j' \) denote truthful bidding and untruthful bidding, respectively. We have \( \frac{p_j'}{\sqrt{d_j}} \geq \max_{k \in B_j^-} \left( \frac{p_k}{\sqrt{d_k}} \right) \geq \frac{p_j}{\sqrt{d_j}} \), otherwise SU \( j \) still cannot win the auction. In this case, its utility can be proved to be non-positive:

\[
\hat{U}_j = v_j - q_j' = v_j - \sqrt{d_j} \times \max_{k \in B_j^-} \left( \frac{p_k}{\sqrt{d_k}} \right) \leq v_j - \sqrt{d_j} \times \frac{p_j}{\sqrt{d_j}} = v_j - p_j = 0 \tag{13}
\]

In summary, SU \( j \) cannot increase its utility by bidding any other value than \( v_j \). In other words, bidding truthfully is a dominant strategy for each buyer.

IV. **Second stage: Recall-based Pricing Game**

In this section, we study the impacts of spectrum recall on both POs’ and SUs’ utilities based on the auction output from the first stage. We first formulate such strategy decision process as a Stackelberg game, and then analyze the Nash equilibrium (NE) of this recall-based pricing game.
A. Game Model Formulation

We assume that POs and SUs are intelligent in our proposed framework. Each PO first announces a maximum quantity of spectrum that may be recalled, and then each SU determines a payment reduction in order to maximize its utility under this recall-based system.

We can formulate the strategy decision process as a Stackelberg game, in which POs act as leaders and SUs play as followers. The leader selects the optimal strategy based on the knowledge of its effect on the followers’ actions. For winning SUs, they have been assigned their desired spectrum in the first stage. However, they are informed of a potential spectrum recall by their allocated POs. Thus, SUs within one PO would compete with each other in order to decrease the amount of recalled spectrum from themselves. This results in a noncooperative payment reduction game, where pricing scheme can be used to adjust the amount of spectrum recalled on SUs according to their payments.

The NE of the proposed recall-based pricing game is solved by backward induction. Namely, we first derive the NE of the game among SUs given the quantity of recalled spectrum and then calculate the best responses of POs.

B. Utility Functions of POs and SUs

Each PO and its assigned winning SUs can be formed in one group since the winner assignment has been done in the first stage. Thus, each PO could run its Stackelberg game independently in its own group and its spectrum recall strategy would not affect the other users in other groups. From this observation, we can focus on one PO only and omit the subscript $i$ in the following context for notation simplicity.

1) Recall mechanism: Let $\Theta$ be the set of winning SUs allocated to a PO. Then, the total spectrum bandwidth assigned to $\Theta$ after the first stage is $T = \sum_{j \in \Theta} d_j$. We define that the maximum quantity of spectrum recalled from $\Theta$ declared by the PO is

$$ RC = \omega T = \omega \sum_{j \in \Theta} d_j, \quad \omega \in [0, 1] \tag{14} $$

where $\omega$ represents the percentage of spectrum recalled from $T$, which is declared by the PO at the beginning of the second stage.

Thus, the PO actually reserves $W - (1 - \omega)T$ spectrum for its own users. Though the PO would compensate SUs in $\Theta$ for the spectrum recall, this behavior violates the contract made in the first stage. As a response, each winning SU $j$ could determine a parameter $\beta_j \in [0, 1]$, so that the actual payment of SU $j$ is reduced to $\beta_j q_j$. Let $r_j$ be the quantity of spectrum recalled from SU $j$. Then, if $r_j = d_j$, all winning spectrum of SU $j$ would be recalled so that its utility turns to be zero or $\beta_j = 0$. By considering the fairness on spectrum recall, we define $r_j$ as

$$ r_j = RC \times \left( \frac{1/\varepsilon_j}{\sum_{k \in \Theta} 1/\varepsilon_k} \right), \quad \forall j \in \Theta. \tag{15} $$

where $\varepsilon_j$ indicates the actual unit payment of SU $j$ and can be calculated as

$$ \varepsilon_j = \frac{\beta_j q_j}{d_j}, \quad \forall j \in \Theta. \tag{16} $$

According to the definitions of (15) and (16), $r_j$ is inversely proportional to the actual unit payment declared by each SU. We assume that $d_j \geq r_j, \forall j \in \Theta$, so that all winning SUs are willing to follow this spectrum recall mechanism. In fact, this assumption can be relaxed unless the total quantity of spectrum recall is larger than a certain threshold $\Gamma$ (which is further explained in Section V). Furthermore, the recall mechanism defined in (15) is also regarded as a common knowledge known to all users.

2) Utility Function of the Winning SUs: With spectrum recall, the transmission rate of SU $j$ can be rewritten as

$$ G_j = (d_j - r_j) \log_2 (1 + \eta_j), \quad \forall j \in \Theta. \tag{17} $$

where $d_j - r_j$ denotes the bandwidth that SU $j$ can actually obtain through its payment of $\beta_j q_j$.

In this recall-based system, the goal of all winning SUs is to prevent their transmission rates from experiencing significant degradation, while at the same time, lower their payments in order to reduce the risk from spectrum recall. Obviously, equation (17) is only a function of $r_j$ because all other parameters are fixed. Since $r_j$ is decided by the parameter $\beta_j$, the strategy of SU $j$ is actually the determination of $\beta_j$. The change on $\beta_j$ would ultimately lead to an impact on PO’s spectrum recall distribution on its winners, and in turn determine the achievable rate of SU $j$.

POs may recall their leased spectrum to deal with their own users’ demand peak during the transmission period. Thus, the services of winning SUs would be degraded. We can formulate the compensation function as

$$ H_j = \xi_j r_j \tag{18} $$

where $\xi_j$ represents the reported compensation index and can be calculated by SU $j$ as

$$ \xi_j = \kappa_j \log_2 (1 + \eta_j) \tag{19} $$

Here $\kappa_j$ is defined as a coefficient of compensation rate for SU $j$ and it also reflects the SU’s attitude towards recalling. Note that $\kappa_j$ is a parameter pre-determined by the system based on the user’s service requirement, so that it cannot be changed arbitrarily in the game.

With all the above settings, we could formulate the utility function of SU $j$, called $U^*_j$, which includes the revenue through its achievable transmission rate, actual payment amount, and the monetary compensation for spectrum recall.

The expression of $U^*_j$ can be presented as

$$ U^*_j = \sigma_j G_j(\beta_j) - \beta_j q_j + H_j \tag{20} $$

where $\sigma_j G_j(\beta_j)$ is the valuation of the transmission rate actually obtained by SU $j$, which is also a function of the variable $\beta_j$. Substitute (18) and (19) into (20), the utility function of SU $j$, $\forall j \in \Theta$, could be rewritten as

$$ U^*_j = [\sigma_j d_j - (\sigma_j - \kappa_j) r_j] \log_2 (1 + \eta_j) - \beta_j q_j \tag{21} $$
where \( \sigma_j - \kappa_j \geq 0 \), which indicates that \( U^t_j \) should not be increased with the quantity of recalled spectrum, so that winning SUs would compete for reducing its \( r_j \).

3) Utility Function of the PO: For the PO, we represent the spectrum bandwidth available for its own PUs as \( S \). Obviously, \( S \) consists of the quantity of both unleased and recalled spectrum, i.e., \( S = W - T + RC \). Thus, the maximum number of PUs that PO could accommodate is

\[
F = \left[ \frac{S}{s} \right] = \left[ \frac{W - (1 - \omega) \sum_{j \in \Theta} d_j}{s} \right]
\]

where \( s \) is the spectrum demand of each PU. By considering \( F \) as the number of servers in an \( M/M/c \) queueing system, i.e., \( c = F \), the mean waiting time for arriving PUs can be calculated as [30]:

\[
M_w = \frac{C(F, \rho)}{\mu(F - \rho)}
\]

where \( \rho = \lambda / \mu \) denotes the utilization factor and \( C(F, \rho) \) is the queueing probability as

\[
C(F, \rho) = \frac{\rho^F / F!}{[\rho^F / F! \sum_{k=0}^{F-1} (\rho^k / k!)] + \rho^F / F!}
\]

According to our system model, PUs has a QoS requirement that the mean waiting time should be not longer than \( \alpha \), otherwise a penalty would be introduced in the utility function of the PO. However, the ultimate goal of PO is to maximize its total utility including the revenue of its own users’ service and the economic profit from spectrum leasing. Therefore, depending on the penalty and profit, the PO may not always try to keep \( M_w \) being less than \( \alpha \) so as to maximize its overall benefits.

If \( \omega = 0 \), the PO will not enter the second stage since there is no need to build a recall-based pricing game. Hence, we only focus on the case when \( \omega \in (0, 1) \) and \( M_w \geq \alpha \). Under this situation, the revenue of the PO gained from its own users, \( \chi \), could be expressed as

\[
\chi = Fu - \Lambda(\psi, M_w, \alpha)
\]

where \( u \) is the individual utility of the PU, \( \Lambda(\cdot) \) denotes the penalty function and \( \psi \) represents the weight index of the penalty for QoS degradation.

The utility of PO consists of three terms, i.e., revenue from its PUs, profits from spectrum leasing, and the compensation caused by its spectrum recall. Thus, we can define the utility function of PO as

\[
U^P = \chi(\omega) + \sum_{j \in \Theta} \beta_j q_j - \sum_{j \in \Theta} H_j
\]

where \( \chi(\omega) \) represents the revenue function of \( \omega \). Therefore, each PO could maximize its own utility by choosing the best strategy of \( \omega \).

C. Nash Equilibrium of the Game among SUs given \( \omega \)

Given \( \omega \), SUs in \( \Theta \) would compete with each other to maximize their utilities by selecting their own strategy \( \beta \). Let \( t \) denote the number of SUs in set \( \Theta \), i.e., \(|\Theta| = t\), and rearrange the indices of these SUs from 1 to \( t \). Then, a noncooperative pricing game could be denoted as \( G_{su} = \{ t, \{ P_j \}, \{ U^t_j(\cdot) \} \} \), where \( P_j \) and \( U^t_j(\cdot) \) are the strategy set and the utility function of SU \( j \), respectively. We further assume that \( t > 1 \) (Otherwise, there is no competition among SUs or the game does not exist).

In this game, each SU \( j, 1 \leq j \leq t \), selects its strategy \( \beta_j \) to maximize its utility \( U^t_j(\beta_j, \beta_{-j}) \), where \( \beta_{-j} \) represents the strategy of all other SUs in \( \Theta \). Then, the Nash equilibrium of the game could be defined as follows.

Definition 4.1: A strategy profile \( \beta = (\beta_1, \beta_2, \ldots, \beta_t) \) is a Nash equilibrium (NE) of the game \( G_{su} = \{ t, \{ P_j \}, \{ U^t_j(\cdot) \} \} \) if for every user \( j, U^t_j(\beta_j, \beta_{-j}) \geq U^t_j(\beta'_j, \beta_{-j}) \) for all \( \beta'_j \in P_j \).

Next, we will prove the existence and uniqueness of NE, and then calculate the unique NE point of the game.

1) Existence of NE:

Theorem 4.1: The game \( G_{su} = \{ t, \{ P_j \}, \{ U^t_j(\cdot) \} \} \) has at least one NE.

Proof: Since \( \beta_j \in [0, 1] \), it is obvious that \( P_j \) is a nonempty, convex and compact subset of the Euclidean space \( \mathbb{R}^n \).

According to the utility function of SU \( j \) in (21), it is not difficult to find that \( U^t_j \) is continuous. Take the first order derivative of \( U^t_j \) with respect to \( \beta_j \). We have

\[
\frac{\partial U^t_j}{\partial \beta_j} = -(\sigma_j - \kappa_j) \log_2(1 + \eta_j) \frac{\partial r_j}{\partial \beta_j} - q_j
\]

(27)

With the definition of \( r_j \) in (15) and (16), we have

\[
\frac{\partial r_j}{\partial \beta_j} = -\frac{RC \cdot J_{1,j} J_{2,j}}{(1 + J_{1,j} J_{2,j} \beta_j)^2}
\]

(28)

where \( J_{1,j} = q_j / d_j \) and \( J_{2,j} = \sum_{k \neq j} \frac{d_k}{\eta_k} \).

Substituting (28) to (27), the second order derivative of \( U^t_j \) can be calculated as

\[
\frac{\partial^2 U^t_j}{\partial \beta_j^2} = -(\sigma_j - \kappa_j) \log_2(1 + \eta_j) \frac{RC \cdot J_{1,j} J_{2,j}}{(1 + J_{1,j} J_{2,j} \beta_j)^3} \leq 0
\]

(29)

The above inequality holds since \( \sigma_j \geq \kappa_j \) and no other terms are less than 0. Therefore, the second derivative of \( U^t_j \) is always less than or equal to 0, which means that \( U^t_j(\cdot) \) is concave in its strategy space.

Thus, \( G_{su} \) has at least one NE since the game \( G_{su} \) has a nonempty, convex and compact strategy space \( P_j \), and \( U^t_j(\cdot) \) is continuous and concave, according to [31].

2) Uniqueness of NE:

Theorem 4.2: The game \( G_{su} = \{ t, \{ P_j \}, \{ U^t_j(\cdot) \} \} \) has an unique NE.

Proof: Let \( \delta_j(\beta) \) be the best response function of SU \( j \). We first check whether \( \delta_j(\beta) \) is a standard function.

Given the utility function of SU \( j \) in (21), the best response \( \delta_j(\beta) \) could be obtained by solving the following equation:

\[
\frac{\partial U^t_j}{\partial \beta_j} = (\sigma_j - \kappa_j) \log_2(1 + \eta_j) \frac{RC \cdot J_{1,j} J_{2,j}}{(1 + J_{1,j} J_{2,j} \beta_j)^2} - q_j = 0
\]

(30)

To simplify the formula, let \( J_{3,j} = (\sigma_j - \kappa_j) RC \log_2(1 + \eta_j) \frac{RC \cdot J_{1,j} J_{2,j}}{(1 + J_{1,j} J_{2,j} \beta_j)^3} \leq 0 \).
Then, the solution of (30) could be derived as

\[ \delta_j(\beta) = \sqrt{\frac{J_{3,j}}{q_j J_{1,j} J_{2,j}}} - \frac{1}{J_{1,j} J_{2,j}} \]  

(31)

Obviously, the term \( \frac{J_{3,j}}{q_j J_{1,j} J_{2,j}} \) is greater than 0. Furthermore, since the strategy space is defined in \([0, 1]\), the existence of solution needs the following constraint:

\[ 1 \leq \frac{J_{1,j} J_{2,j} - J_{3,j} q_j}{q_j} \leq (J_{1,j} J_{2,j} + 1)^2 \]  

(32)

Since \( \delta_j(\beta) \) is obviously a quadratic function, \( \delta_j(\beta) \) would be monotonically increasing when \( \frac{\partial \delta_j(\beta)}{\partial \beta} \geq 0 \). Calculate the derivative with respect to \( \beta \) as

\[ \frac{\partial \delta_j(\beta)}{\partial \beta} = \frac{-1}{\sqrt{J_{1,j} J_{2,j}}} \left( \frac{1}{J_{1,j} J_{2,j}} + \frac{1}{J_{1,j} J_{2,j}} \right) \]  

(33)

Since from (32), we have

\[ J_{2,j} \geq \sqrt{\frac{4q_j}{J_{1,j} J_{3,j}}}, \]  

(34)

we can easily prove that \( \frac{\partial \delta_j(\beta)}{\partial \beta} \geq 0 \), i.e., \( \delta_j(\beta) \) is a monotonically increasing function.

We then calculate \( \phi \delta_j(\beta) - \delta_j(\phi \beta) \) as:

\[ \phi \delta_j(\beta) - \delta_j(\phi \beta) = \phi - \sqrt{\frac{J_{1,j} J_{2,j} J_{3,j} - 1}{q_j J_{1,j} J_{2,j}}} - \phi \sqrt{J_{1,j} J_{2,j} J_{3,j} - 1} \]  

(35)

For \( \forall \phi > 1, \phi - \sqrt{\phi} > 0 \). The above equation is always positive, which means that \( \phi \delta_j(\beta) - \delta_j(\phi \beta) > 0 \) and \( \delta(\beta) \) is scalable.

Since the best-response \( \delta_j(\beta) \) is proved to be positive, monotonic and scalable, according to [32], it is a standard function. From [33], we know that the game \( G_{su} \) with \( \delta_j(\beta) \) as a standard function has an unique NE.

3) The NE point of the game \( G_{su} \). For a noncooperative game, NE is defined as the operation point(s) at which no player could improve the utility by changing its strategy unilaterally. Since the NE of the game \( G_{su} \) has been proved to be existing and unique, we could derive the unique NE point \( \beta_j^* \) by solving the following equation set [31]:

\[ \beta_j^* = \sqrt{\frac{J_{3,j}}{q_j J_{1,j} \sum_{k \neq j} \frac{d_j}{q_k \beta_k} - \frac{1}{J_{1,j} \sum_{k \neq j} \frac{d_j}{q_k \beta_k}}}, \quad \forall j \in \Theta \]  

(36)

Although the above equations are not difficult to be solved, deriving a closed-form expression is not easy. Since the best response of the leader (PO), i.e., \( \omega^* \), can only be obtained by substituting the NE point of the \( G_{su} \) into the PO’s utility function, we try to express \( \beta_j^* \) in terms of \( \omega \) as follows.

Consider the case with only 2 winners, i.e., \( |\Theta| = t = 2 \).

According to (36), the equation set for the NE point becomes

\[ \begin{align*}
\beta_1 = & \sqrt{\frac{B_1}{A_1 + q_j \beta_j^*}} - \frac{1}{A_1 + q_j \beta_j^*} \\
\beta_2 = & \sqrt{\frac{B_2}{A_2 + q_j \beta_j^*}} - \frac{1}{A_2 + q_j \beta_j^*}
\end{align*} \]

(37)

where \( A_j = J_{1,j} \) and \( B_j = J_{3,j}/q_j \).

After some simple manipulations, we have

\[ \begin{align*}
\beta_j^* = & \sqrt{\frac{B_j}{A_j + q_j \beta_j^*}} + \frac{1}{A_j + q_j \beta_j^*} \\
\beta_j^* \beta_j^* - \beta_j^* = & \frac{B_j}{A_j + q_j \beta_j^*} + \frac{1}{A_j + q_j \beta_j^*}
\end{align*} \]

(38)

By simple observation, we could find out that \( \beta_1^* \) and \( \beta_2^* \) satisfy

\[ \beta_2^*/\beta_1^* = B_1/B_2 \]

(39)

Therefore, the NE is

\[ \begin{align*}
\beta_1^* = & \frac{A_1 J_{3,j} C_j}{(A_1 J_{3,j} + A_2 J_{3,j})} \omega \\
\beta_2^* = & \frac{A_2 J_{3,j} C_j}{(A_1 J_{3,j} + A_2 J_{3,j})} \omega
\end{align*} \]

(40)

For \( t > 2 \), because of the symmetry property of the equation set, the general solution can be represented as

\[ \beta_j^* = \Pi_j \omega, \quad \forall j \in \Theta \]

(41)

where \( \Pi_j \) is a coefficient associated with SU \( j \). For example, when \( t = 2 \), \( \Pi_1 = \frac{A_1 J_{3,j} C_j}{(A_1 J_{3,j} + A_2 J_{3,j})} \) and \( \Pi_2 = \frac{A_2 J_{3,j} C_j}{(A_1 J_{3,j} + A_2 J_{3,j})} \).

D. Best Response of the PO

Based on the strategies making by the followers (SUs), the leader (PO) can then calculate its best response \( \omega^* \) as follows.

By substituting (38) into (26), \( U^p \) can be expressed as

\[ U^p = \chi(\omega) - \omega \sum_{j \in \Theta} \kappa_j T \log_2 (1 + \eta_j) \sum_{j \in \Theta} \frac{d_j}{\eta_j} - \omega \sum_{j \in \Theta} \Pi_j q_j \]  

(42)

Let \( \vartheta = \sum_{j \in \Theta} \kappa_j T \log_2 (1 + \eta_j) \frac{d_j}{\eta_j} - \sum_{j \in \Theta} \Pi_j q_j \).

Equation (42) can be rewritten as

\[ U^p = \chi(\omega) - \omega \vartheta \]

(43)

Unfortunately, directly calculating the derivative of \( \chi(\omega) \) with respect to \( \omega \) is difficult, because it is hard to build the penalty function, \( \Lambda(\cdot) \), based on (23) and (24). For explanation purpose, in this paper, we define \( \Lambda(\cdot) \) as a sigmoid function of \( \omega \). In fact, as \( \omega \) increases, each PO can recall more spectrum to serve its own PU s so that the waiting time of arriving PUs will decrease. Thus, \( \Lambda(\cdot) \) should be a decreasing function of \( \omega \). However, such decreasing trend should not be linear. Intuitively, when the amount of spectrum reserved by the PO is much less than the required amount of spectrum to ensure the desired QoS, the increase of \( \omega \) will result in
a significant improvement on the QoS. However, when the amount of reserved spectrum is close to the required amount, the effect of increasing $\omega$ becomes gradually. Obviously, such observation can be well depicted by a sigmoid function. Note that sigmoid function has been widely used in literature to formulate users’ satisfaction with respect to service quality or resource allocation [34]–[36]. Specifically, the definition of $\Lambda(\cdot)$, i.e., PUs’ degree of dissatisfaction to their QoS, is

$$
\Lambda(\psi, M_e, \alpha, \omega) = \frac{1}{1 + e^{\psi - \omega(M_e - \alpha)}}
$$

(41)

where $\alpha$ denotes the QoS requirement of its PUs, $\psi$ indicates the weight index of penalty for QoS degradation and $M_e$ is the current waiting time of the system when spectrum recall is not enabled. Obviously, $\Lambda(\psi, M_e, \alpha, \omega)$ is a decreasing function of $\omega$. We could now rewrite the revenue function $\chi(\omega)$ as

$$
\chi(\omega) = F(\omega)u - \frac{1}{1 + e^{\psi - \omega(M_e - \alpha)}}
$$

(42)

where $F(\omega)$ is the number of PUs that the PO could accommodate under such circumstance.

Let $\nu = \psi^{-1}(M_e - \alpha)$ and take the first order derivative of $U^p$ in (40) according to $\omega$. We have

$$
\frac{\partial U^p}{\partial \omega} = \frac{\nu e^{\psi \omega}}{(1 + e^{\psi \omega})^2} + \frac{T}{s}u - \vartheta
$$

(43)

Since $x/(x + 1)^2 \leq 1/4$ for any $x > 0$ (it can be easily proved based on the observation that the left-hand side of this inequality reaches the maximum when $x = 1$), we have the following inequality

$$
\frac{\nu e^{\psi \omega}}{(1 + e^{\psi \omega})^2} + \frac{T}{s}u - \vartheta \leq \frac{\nu}{4} - \left(\vartheta - Tu/s\right)
$$

(44)

If the right-hand side of (44) is negative, we have $\frac{\partial U^p}{\partial \omega} < 0$ and $\omega = 0$ yields the maximum utility for PO. If $\nu/4 - (\vartheta - Tu/s) \geq 0$, with the increase of $\omega$, $U^p$ would first decrease, then increase, and finally decrease again. In order to find the point of $\omega$ which results in the maximum $U^p$, let $\frac{\partial U^p}{\partial \omega} = 0$ and calculate $\omega$ as

$$
\omega = \frac{1}{\nu} \ln \left(\frac{-\nu + \sqrt{\nu^2 - 4\nu(\vartheta - Tu/s)}}{2(\vartheta - Tu/s)}\right)
$$

According to the trend of function $U^p$, it is not difficult to figure out that, if $\omega_1 < \omega_2$, the maximum value of $U^p$ would be achieved at $\omega_2$. Hence, we finally obtain the best response of $\omega$ as (45).

The Stackelberg game will then update the NE of the pricing game $G_{su}$ by substituting $\omega^*$ into (36). At the end, each PO $i$ would recall $\omega^* T_i$ spectrum in order to maximize its total utility and each winning SU $j$, $j \in \Theta_i$, would decrease its payment to $(1 - \beta^*_tf_j)q_j$ so as to offset its risk in spectrum recall.

V. PERFORMANCE ANALYSES OF TAGS

In this section, the economic properties of TAGS are analyzed in terms of incentive compatibility and individual rationality. The detailed procedure of TAGS is also presented.

A. Economic Properties

Since TAGS consists of two sequential stages, we need to re-examine that the overall TAGS could produce an economically feasible solution.

Theorem 5.1: TAGS is incentively compatible.

Proof: For each SU, its strategies are its bidding in the first stage and its payment reduction in the second stage. Lemma 3.2 has proved that truthful bidding is a dominant strategy for each SU in the first stage. In addition, the decision process of payment reduction in the second stage is indeed a complete information game where incentive compatibility is not a concern. Hence, all SUs would not cheat in TAGS.

For each PO, the quantity of spectrum recall is determined by the NE of Stackelberg game in the second stage. Thus, we only need to examine whether each PO would like to auction all its idle spectrum in the first stage. Since each PO $i$ is enabled to recall spectrum to satisfy the potential increasing demand from its own PUs, it takes no risk on balancing the amount of spectrum for leasing and reservation. Therefore, the amount of auctioned spectrum $C_i$, which results in a maximum auction revenue $I_i$, could also lead to a maximum utility for PO $i$, i.e.

$$
\arg \max_{C_i} U^p_i = \arg \max_{I_i} I_i, \quad \forall i \in M
$$

(46)

Since the auction revenue $I_i$ is obviously a non-decreasing function of $C_i$, in order to maximize $U^p_i$, PO $i$ should auction all its idle spectrum in the first stage. Therefore, both buyers and sellers would bid truthfully in TAGS.

Theorem 5.2: TAGS is individually rational for all sellers, which means that all POs would be guaranteed with non-negative utilities.

Proof: Consider the expression of $U^p$ in (40). In the first stage, each PO makes the contract with the broker and accordingly, the broker calculates the payoff $I_i$ for each PO $i$. However, POs may break the contract by increasing $\omega$ if and only if their utilities could be enhanced. Since POs are leaders in the Stackelberg game, which means that they are aware of all the information, we have

$$
U^p_i \geq I_i \geq 0, \quad \forall i \in M
$$

(47)

Hence, the utilities of all POs in TAGS must be larger than or equal to 0.

Theorem 5.3: TAGS provides non-negative utilities for all winning SUs in $\Theta_i$ when the spectrum recall ratio $\omega_i$ is no more than a certain threshold $\Gamma_i$.  


Proof: First, let’s consider the case when \( \omega_i = 0 \). In this case, the utility of each SU \( j \) is only determined by the spectrum auction, which has already been proved to be non-negative in Lemma 3.1.

Consider the utility function of SUs in (21) when \( \omega_i \neq 0 \). In order to ensure the utility of SU \( j \) to be non-negative, the sum of the compensation and payment reduction should be greater than or equal to the utility loss caused by QoS degradation, i.e.,

\[
(1 - \beta_j)q_j \geq (\sigma_j - \kappa_j)r_j \log_2(1 + \eta_j), \quad \forall j \in \Theta_i
\]

By substituting \( \beta_j \) at the NE point in (38) and the recall distribution \( r_j \) in (15), we have

\[
(1 - \Pi_j \omega_i)q_j \geq (\sigma_j - \kappa_j)\omega_i T_i \frac{d_j}{\Pi_j q_j} \log_2(1 + \eta_j) \quad (49)
\]

Let \( J_{4,j} = (\sigma_j - \kappa_j)T_i \frac{d_j}{\Sigma_{k \in \Theta_i} d_k q_k} \log_2(1 + \eta_j) \), the above inequality finally yields:

\[
\omega_i \leq \frac{q_j}{J_{4,j} + \Pi_j q_j}, \quad \forall j \in \Theta_i
\]

Therefore, we could draw a conclusion that when \( \omega_i \leq \Gamma_i = \min\{ J_{4,j} + \Pi_j q_j, \forall j \in \Theta_i \} \), all SUs in \( \Theta_i \) could be guaranteed with non-negative utilities.

Theorem 5.3 implies that the maximum amount of spectrum recall declared by PO \( i \) should be \( \min \{ \omega_i^*, \Gamma_i \} \) so as to maintain the rationality of auction mechanism while maximizing its own utility.

B. Detailed Procedure of TAGS

The procedure of TAGS is summarized as follows:

- At the beginning of each frame, all POs and SUs report their bids to the spectrum broker.
- The spectrum broker collects the received sealed information and then sets up a combinatorial spectrum auction. The winners determination and payment scheme follows the approximation mechanism designed in Section III-B.
- After the auction, each PO informs its own winning SUs a maximum quantity of spectrum recalled and along with its recall scheme as in (15). Each SU then decides a payment reduction to offset the risk while at the same time trying to avoid large utility degradation. POs in turn derive the spectrum recall ratio distributed on each winning SU when the payment strategies have been finally determined.
- In the rest time of each frame, POs would recall spectrum to meet the increase of their own spectrum demand from newly arrived PUs. The total amount of spectrum recalled by each PO and the distributed recall ratio on each winner follow the strategies made in the previous step. At the end of each frame, POs compensate each winner based on (18).

VI. NUMERICAL RESULTS

In this section, simulations are conducted to evaluate the proposed spectrum allocation algorithm TAGS. The performance of two stages is first illustrated separately and then the overall performance of TAGS in terms of spectrum utilization and utilities is presented.

A. Simulation Settings

Consider a CR network with \( m \) POs and \( n \) SUs randomly scattered in a \( 200 \times 200 \) geographic area, where \( m \) varies from 5 to 50 and \( n \) is 100. Each PO \( i \) owns a total spectrum bandwidth \( W_i \) randomly in \([20, 100]\) MHz and a same auction radius \( R = 20, 50 \) or 80. Each primary user has the same demand \( s \) which is selected as 1, 2 or 3 MHz. The activities of PUs among different POs are heterogeneous with arrival rate \( \lambda_i = 1, 2 \) or 3 and service rate \( \mu_i \) randomly selected from 0.1 to 0.2. For simplicity, assume that all PUs have the same waiting time requirement \( \alpha = 6.25 \times 10^{-4} \) s and the same individual utility \( u = 2 \times 10^5 \). The spectrum demand \( d_j \) and the SNR \( \eta_j \) of each SU \( j \) are randomly selected in \([1, 10]\) MHz and \([100, 200]\), respectively. Furthermore, the weight indices are defined as follows: \( \psi_i = [1, 2, 3] \) for each PO \( i \); \( \sigma_j = 1 \) and \( \kappa_j \) is randomly chosen in \([0, 1]\) for each SU \( j \). Suppose that the length of each frame \( T = 6 \) s and we observe all results from 150T to 200T. Note that some parameters may vary according to evaluation scenarios.

B. Performance at the First stage

Fig. 3 shows the social welfare (averaged from 150T to 200T) achieved by our proposed algorithm for winner determination in the spectrum auction. For comparison purpose, the pure allocation (PA) is also simulated as the benchmark, which iteratively selects the unallocated SU with largest spectrum demand and assigns it to POs regardless of its bidding price. It shows that our proposed algorithm could achieve much higher social welfare than PA and this superiority becomes more obvious when the number of POs increases. The reason is that the larger number of POs enhances the probability for better spectrum allocation. Moreover, by comparing with the initial solution of Algorithm 1, we could clearly observe that our proposed improvement processes can effectively improve the social welfare. By further considering its polynomial
computational complexity, the proposed auction mechanism is more feasible for practical applications.

Fig. 4 illustrates the impact of the POs’ auction radius on social welfare. It is apparent that the larger trading radius POs have, the higher social welfare the first-stage algorithm would achieve. The explanation is twofold: i) The number of covered SUs is increased with the auction radius, which results in more competitors in the spectrum auction; ii) The larger area of overlapped trading regions enhances the effectiveness of our proposed solution improvement processes by increasing the probability for local exchanges. Furthermore, Fig. 4 shows that the gap between the curves of $R = 80$ and $R = 50$ is less than the one between $R = 50$ and $R = 20$. That is because when the radius continues to increase, the effect due to an increased number of competitors becomes less obvious.

**C. Performance at the Second stage**

To investigate the properties of the second stage, we fix both the number of POs and their auction radius to be 50 without loss of generality.

In Fig. 5, the properties of the NE in $G_{su}$ are examined. For concise and clear demonstration, the situation with $t = 2$ is considered. Fig. 5 shows that for any value of recall ratio $\omega$, there is a unique intersection point between the best responses of SU 1 and SU 2. Note that according to our analysis in Section IV-B1, $\beta_1 \neq 0$ and $\beta_2 \neq 0$. Otherwise, (15) becomes meaningless. In addition, the points of NE under different recall ratios locate on a straight line, which verify the calculation in (37) that the ratio between $\beta_1^*$ and $\beta_2^*$ is always a constant, i.e., $\frac{\beta_1^*}{\beta_2^*} = \frac{B_1}{B_2}$.

Fig. 6 reveals a PO’s utility with the increase of its spectrum recall ratio. Given the NE of $G_{su}$, the PO could take initiative by deciding $\omega$ so as to maximize its utility $U^p$. The trend of these curves indicates that $U^p$ first increases with $\omega$. That is because with the increase of $\omega$, more spectrum is available for recall so that more utility from PUs and less penalty for QoS degradation are achieved. However, after a certain point, since the compensation to winning SUs becomes dominant, $U^p$ decreases. The $\omega$ which results in the highest utility is the best response. We could also observe from the figure that a larger value of penalty weight $\psi$ results in a larger value of the best response for PO. This is because the spectrum requirement from PUs increases when the penalty becomes larger. As a consequence, PO would prefer to recall more spectrum to satisfy its own users’ demands.

**D. Overall Performance of TAGS**

Fig. 7 compares the spectrum utilization ratio (i.e., the ratio between the amount of occupied spectrum and the total amount of POs’ spectrum) with and without the proposed TAGS. When TAGS is not applied, both spectrum auction and recall pricing game are not activated so that such ratio is only determined by the spectrum usage of PUs. Nearly half of the spectrum is under-utilized in this circumstance whereas the spectrum demands from PUs could be fully satisfied. On the contrary, with the employment of TAGS, a balance on spectrum utilization between SUs and PUs can be reached so as to achieve a much higher utilization ratio.

The actions of a randomly picked PO during 150T and 200T are shown in Fig. 8. The curve of increasing spectrum demand from PUs within each frame is plotted. However, due to the Stackelberg game formulated in the second stage, we
utility of SUs in winner set $\Theta_i$ with the change of PO $i$’s recall ratio in Fig. 9. It is shown that the curve is slowly decreasing with the increase of $\omega_i$. Note that the curve is nonlinear since it does not represent the utility of one specific SU but the minimum utility of SUs in $\Theta_i$. Furthermore, there exists a threshold of $\Gamma_i$ to guarantee that the minimum SU’s utility is greater than or equal to zero, and thus ensure all winning SUs to have non-negative utilities.

Fig. 10 shows the superiority of TAGS on the utility of a randomly selected PO. Without TAGS, the PO’s utility, $U^p$, only includes its own PUs’ revenue. According to the analysis of Fig. 7, since plenty of spectrum is under-utilized without running spectrum auction, $U^p$ is relatively low. It can be also seen from the figure that the PO has highly fluctuating utility without the second stage of TAGS. This is because even though more utility is produced from the first-stage auction, the increasing demand from its own PUs leads to a large penalty on $U^p$ due to QoS degradation. In summary, the curve with both stages of TAGS apparently shows the best performance since spectrum recall is enabled and the best recall quantity is determined from the Stackelberg game in the second stage.

VII. CONCLUSIONS

In this paper, spectrum sharing among multiple heterogeneous POs and SUs in recall-based cognitive radio networks has been discussed. To address the complexity of the system model, we propose a two-stage scheme, called TAGS, which consists of a geographically restricted combinatorial spectrum auction for initial spectrum allocation and a Stackelberg game for deciding best strategies towards potential spectrum recall. Theoretical and simulation results show that the proposed scheme TAGS can improve the utilities of POs, enhance the spectrum efficiency and provide economic incentives for all users to participate in spectrum sharing.

REFERENCES
