Spectrum Auction for Differential Secondary Wireless Service Provisioning with Time-Dependent Valuation Information

Changyan Yi, Student Member, IEEE, Jun Cai, Senior Member, IEEE, and Gong Zhang

Abstract—In this paper, we propose a spectrum auction mechanism for secondary spectrum access in cognitive radio networks. Different from existing works in the literature, the time-dependent buyer valuation information is employed in the proposed mechanism so that the primary spectrum owner (PO) can determine more favorable spectrum allocations and pricing functions in order to maximize the expected auction revenue. In addition, to exploit the temporal spectrum reusability, the proposed mechanism allows each secondary wireless user to declare its specific time preferences including service starting time, delay tolerance and service length. By further considering the heterogeneities in secondary wireless service provisioning, the proposed mechanism is able to support heterogeneous forms (continuous or disjointed spectrum usages) of secondary spectrum requests. Specifically, at the beginning of the auction frame, secondary wireless users report their different spectrum usage requests along with the bidding prices, while the PO decides a single-step spectrum allocation and calculates the payment for each winner based on not only the received bids but also the known time-related information. The theoretical analyses and simulation results show that the proposed auction mechanism can satisfy all desired economic properties, and can improve the spectrum allocation efficiency and auction revenue compared to counterparts.

Index Terms—Secondary spectrum access, cognitive radio networks, auction mechanism, time-dependent valuation information, differential wireless service provisioning.

I. INTRODUCTION

Radio spectrum is a scarce and precious resource in wireless networks due to the tremendous growth of new wireless devices, applications and services. Current spectrum regulatory policy employs static allocation patterns, leading to significantly inefficient spectrum utilization [1]. To address this issue, cognitive radio (CR) based dynamic spectrum access (DSA) [2]–[4] has been proposed to redistribute spectrum in a more intelligent and flexible way. Among various methods for the implementation of DSA, market-driven spectrum sharing [5] is one of the promising approaches because of its capability in enhancing the spectrum utilization and producing economic incentives for all participants.

Auction is a well-known market-driven mechanism in constructing economic models for DSA, and has been widely discussed in the literature [6]–[12]. A general goal of spectrum auction design is the satisfaction of the direct revelation principle [13], which requests incentive compatibility, individual rationality, and allocation feasibility. In other words, the auction mechanism should be designed to force all secondary spectrum buyers to behave truthfully (i.e., the best strategy is to report the truthful information), and grant each participant in the auction with a non-negative utility. Another important goal in spectrum auction design is the maximization of auction revenue so as to encourage spectrum owners to join the market.

One unique feature of spectrum auction is the temporal spectrum reusability [14]–[16], i.e., a channel can be allocated to multiple secondary spectrum buyers if their requested service time periods do not overlap with each other. Some existing works on spectrum auction have focused on resolving such a challenge by assuming that secondary spectrum buyers have either no specific time preferences [14] or fixed time requirements [15], [16]. However, in reality, each secondary spectrum buyer may have its preferred service starting time, service length, and delay tolerance. For example, the service of a wireless file transmission is declared at the instant when the request is generated; its service length depends on the transmission rate and the size of the file; and its delay tolerance is determined by the importance of the file to customers. Though spectrum buyers with aforementioned time flexibilities have been taken into account in some recent spectrum auction designs [17]–[19], there are many unaddressed practical issues:

a. The commonly adopted approach in the literature is based on online algorithms [20], which make spectrum allocation decisions myopically along the time.

b. Spectrum buyers are ordinarily required to claim homogeneous service demands (i.e., either continuous or disjointed), while the potential heterogeneities in wireless users’ quality of service (QoS) requirements have rarely been considered.

c. In most of the conventional spectrum auction frameworks [6]–[9], outcomes of spectrum auctions were solely based on the bids from buyers. Recently, more and more studies [21]–[23] in wireless applications revealed that the stochastic information of buyer valuation on radio resources are available to the auctioneer from history. Because of the availability of such extra knowledge, it is possible that the auction revenue could be further improved.

d. It is commonly assumed that buyer valuations on radio resources are independent of the time when these resources...
are occupied. However, in practice, the value of spectrum resources indeed varies with the time. For instance, as reported in [24], the average downstream traffic of cellular networks varies significantly with the time and reaches the peak at 8:00 pm when people arrive at home and start interacting with multimedia. It is intuitive that the increased traffic volume will make the spectrum more precious. By taking this fact into account, it is necessary to design spectrum auctions by considering the situation that buyer valuations depends on the requested service time (i.e., time-dependent buyer valuations).

To address all these challenges, in this paper, we propose a spectrum auction mechanism for heterogeneous secondary wireless service provisioning in CR networks. The primary spectrum owner (PO) who owns multiple spectrum channels acts as the central auctioneer and determines spectrum allocations of its idle channels (i.e., unoccupied channels by its own primary users) among multiple secondary wireless users (SUs). Spectrum requests from all SUs are classified into two classes, i.e., continuous or disjointed spectrum usages in time. In addition, we consider time-dependent buyer valuation information in auction mechanism design. At the beginning of an auction frame, each secondary spectrum buyer reports its bidding price and time preferences in terms of service starting time, delay tolerance and service length. After receiving all bids, the PO decides a single-step spectrum allocation for different classes of secondary spectrum requests and calculates the payment for each winner in order to maximize its expected auction revenue.

However, designing such an auction mechanism is difficult due to the following aspects: 1) introducing time-dependent valuation information will affect both winner and payment determinations, and thus its effects have to be carefully investigated; 2) the heterogeneity on time requirements of secondary spectrum buyers brings more flexibility in spectrum allocation, while at the same time increasing the complexity of finding the optimal solution; and 3) the designed mechanism has to force all users to truthfully reveal not only their private values but also their specific time requirements on spectrum usages. To tackle the above difficulties, in this paper, we first derive an optimal payment rule and then transform the original allocation problem to a dynamic knapsack problem [25] (i.e., a capacity sharing problem for demands with different temporal requirements). Furthermore, since the dynamic knapsack problem is NP-hard, an effective allocation algorithm based on buyer grouping is designed. Finally, we prove that our proposed auction mechanism is economically feasible in terms of incentive compatibility and individual rationality.

The rest of the paper is organized as follows: Section II provides a review of recent related works. Section III describes the system model and presents the problem formulation of the proposed spectrum auction framework. The optimal auction mechanism is designed in Section IV. The analyses of economic properties are presented in Section V. Section VI illustrates the simulation results, and Section VII concludes the paper.

II. RELATED WORKS

Market-driven spectrum auction has been considered as a promising paradigm for CR-based DSA and has received considerable attention in recent years. For example, Khaleedi et al. in [7] proposed a dynamic auction mechanism for secondary spectrum sharing with time-evolving channel qualities. Feng et al. in [8] studied a spectrum auction among secondary wireless service providers, where each of them could flexibly determine the channel demand and corresponding bidding value according to the requirement of its end users. In [6] and [9], Yi et al. considered a new framework of spectrum auction, where the spectrum seller could recall the auctioned spectrum from winning buyers whenever doing so could enhance its own utility. However, all these works assumed that spectrum buyers remained active during the whole auction period and ignored potential temporal re-usage.

Temporal spectrum reusability has been addressed in some existing spectrum auction designs. Zheng et al. in [14] introduced a strategy-proof combinatorial auction for joint spectrum allocation and transmission scheduling, where channels were shared in a time-multiplexing way. In [15], Li et al. developed a truthful and efficient auction scheme in which the spectrum resource was modeled in a time-frequency division manner. However, neither of these works can deal with the scenario that spectrum buyers may have different preferences on service starting time, delay tolerance and service length. To explore time flexibilities, Deek et al. in [17] proposed a truthful online spectrum auction with buyers arriving and departing on the fly. In [18], Sodagari et al. presented a truthful auction mechanism for dynamic spectrum sharing, where the property of collusion-resistance was guaranteed. A location-aware online spectrum auction framework was investigated in [19] which incorporated both buyers’ temporal requirements and location information. However, all these works were restricted in online myopic allocations. Besides, none of these studies discussed the potential heterogeneities in QoS requirements of SUs.

Another common way in modeling resource allocations with dynamic presence of items (or requesters) in time domain
is through the formulation of dynamic knapsack problem. Papastavrou et al. in [25] studied a dynamic and stochastic knapsack problem which aimed to maximize the expected accumulated utility from accepting or rejecting items that arrived according to a Poisson process. Dizzier et al. in [26] described a dynamic knapsack problem for allocating a given capacity of resources to sequentially arriving requesters, and derived an optimal policy which could lead to the maximization of revenue. However, both works cannot be applied in spectrum auctions due to the lack of considerations on the temporal spectrum reusability, i.e., wireless users who request spectrum resources only temporally occupy them during their services and will release them to other users when their services have been completed.

Recently, employing potentially available stochastic buyer valuation information has been widely adopted in auction mechanisms for radio resource allocations. Ayyoub et al. in [21] designed a truthful spectrum auction mechanism under the relaxed Bayesian setting where buyer valuations were drawn from publicly known probability distributions. Huang et al. in [22] analyzed a spectrum auction problem with performance guarantees for social efficiency and auction revenue under available buyer valuation distributions. In [23], an auction mechanism for participatory sensing was constructed, where the cost information of each user followed a known distribution. However, all these works assumed that buyer valuation information was completely independent of the time when radio resource is actually occupied.

In this paper, different from [6]–[9] where static spectrum buyers were considered, we focus on a two-dimensional spectrum auction with heterogeneous time requirements from SUs. Unlike [14]–[16] where time preferences of spectrum buyers were fixed or unrestricted, we allow each spectrum buyer to declare its specific service starting time, delay tolerance and service length. In addition, this work aims to develop a single-step allocation for a whole time frame rather than the online allocations as in [17]–[19]. Moreover, the time-dependent buyer valuation information is adopted in the system model, which makes the auction mechanism design more complicated than [21]–[23]. At last, our work differs from those in [25]–[28] by taking into account the heterogeneous forms of wireless service provisioning.

### III. Model and Problem Description

In this section, we describe the system model under consideration and the problem formulation for the optimal auction mechanism. For convenience, Table I lists some important notations which will be used in this paper.

#### A. System Model

Consider a CR network with multiple heterogeneous SUs and a PO who owns multiple homogeneous orthogonal channels to serve its subscribed PUs. If at some time there are idle channels available, the PO can allow SUs to access these channels in order to obtain some extra profits. Define an auction frame $T = [0, T-1]$, which consists of $T$ uniform time slots, and $M$ as the number of idle channels at the beginning of $T$. As the spectrum auctioneer, the PO has an objective to maximize its expected revenue by leasing its idle channels to different secondary spectrum buyers during $T$. Note that the auction is carried out at the beginning of the frame, rather than each slot.

To reflect the heterogeneity among SUs’ service requests with respect to QoS requirements, we classify them into two categories, i.e.,

- **Class I-SU services**: Each of them requires a discrete spectrum usage during its active period within $T$. For simplicity, we assume that one time-frequency chunk (a single time slot from one channel) is demanded by each request. For the scenario where one service requires multiple time slots, we can treat it as multiple services with single chunk request. A typical example of Class I-SU service is file transfer (such as HTTP/FTP) [15], [18], where continuous service is not required.

- **Class II-SU services**: Each of them requires an immediate spectrum access to one single channel for a demanded period within $T$. Any granted Class II-SU service has to be continuously served without interruption. A typical example of such services is online stream video or audio transmissions.

An example of secondary spectrum usages is illustrated in Fig. 1. We further assume that the network operates in a small region [29], i.e., all SUs are located within the interference

---

**Table I**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>number of time slots in auction frame $T$</td>
</tr>
<tr>
<td>$M$</td>
<td>number of auctioned channels in $T$</td>
</tr>
<tr>
<td>$a_{\ell}$</td>
<td>starting time of each request $\ell$</td>
</tr>
<tr>
<td>$d_{\ell}$</td>
<td>deadline of fulfilling each request $\ell$</td>
</tr>
<tr>
<td>$v_{f}$</td>
<td>unit valuation of each request $f$</td>
</tr>
<tr>
<td>$L_{\ell}$</td>
<td>service length of each request $\ell$</td>
</tr>
<tr>
<td>$G(\cdot)$</td>
<td>function of spectrum gain</td>
</tr>
<tr>
<td>$F_{a}(\cdot)$</td>
<td>PMF of valuation on starting time $a$</td>
</tr>
<tr>
<td>$F_{u}(\cdot)$</td>
<td>CDF of valuation on starting time $a$</td>
</tr>
<tr>
<td>$m^{t}$</td>
<td>number of unallocated channels after $t$ set of requests with starting time as $t$</td>
</tr>
<tr>
<td>$n^{t}$</td>
<td>allocation probability of each request</td>
</tr>
<tr>
<td>$X(\cdot)$</td>
<td>expected unit payment of each request</td>
</tr>
<tr>
<td>$P(\cdot)$</td>
<td>expected service payment of each request unit virtual valuation of each request</td>
</tr>
</tbody>
</table>

---

**Fig. 1.** An illustration of spectrum usages.
range of each other, so that one channel cannot be accessed by multiple users simultaneously. Such assumption can be relaxed by applying users’ clustering methods [30]–[32] for possible spectrum spatial reuse. We leave this in our future works because of the space limitation. Let \( N_1 \) and \( N_2 \) denote the set of requests for Class I and Class II SUs, respectively. The spectrum allocation among SUs can be modeled as an auction, where the PO acts as the auctioneer, and SUs’ service requests can be treated as spectrum buyers.

### B. Types of Secondary Spectrum Buyers

In auction design, the type of a buyer refers to its private information [20]. For a Class I-buyer \( i \in N_1 \), its type can be specified as a 3-tuple \((a_i, d_i, v_i)\), where

- \( a_i \in T \) is the time when buyer \( i \) declares a spectrum service request. In other words, \( a_i \) is the first time slot that buyer \( i \) wants to obtain the service.
- \( d_i \in [a_i, T - 1] \) is the deadline for completing this request or the last time slot that buyer \( i \) still holds a nonzero valuation towards its request. Obviously, \( d_i - a_i \) indicates the delay tolerance of buyer \( i \).
- \( v_i \in V \) is the valuation of buyer \( i \) for receiving the service within its active period \([a_i, d_i]\). For computational tractability, we define \( V \) as a finite set of positive discrete valuations, i.e., \( V = \{\sigma, 2\sigma, \ldots, K\sigma\} \) for any \( \sigma > 0 \), where \( K \) is a finite integer number (which indicates the bound), and \( \sigma \) represents the smallest unit of valuation in the spectrum auction (e.g., 1 cent or 1 dollar in the real market).

For a Class-II buyer \( j \in N_2 \), its service cannot suffer any delay and will last for consecutive time slots. Thus, its type can also be specified by a 3-tuple \((a_j, v_j, L_j)\), where

- \( a_j \in T \) is the requested service starting time.
- \( v_j \in V \) is the unit valuation of buyer \( j \) for receiving a continuous service starting from \( a_j \).
- \( L_j \in [0, T - d_j] \) is the length of the spectrum service or the number of consecutive time slots that buyer \( j \) demands.

In fact, we can depict the type of any buyer \( \ell \in N_1 \cup N_2 \) by using a 4-tuple \((a_\ell, d_\ell, v_\ell, L_\ell)\). Here, \( L_\ell \) = 1 for all \( \ell \in N_1 \), which means that each Class I-ISU service demands a single slot; \( d_\ell = a_\ell \) for all \( \ell \in N_2 \), which indicates that the Class II-ISU service has no delay tolerance; \( v_\ell = v_\ell L_\ell \) for all \( \ell \in N_2 \), which denotes the service valuation of each Class II-ISU request. We limit our auction framework to a similar scenario as in [14], [17], [18], [33], where there are potential misreportings on service starting times, deadlines of fulfillments and valuations, but not on service lengths.

Thus, for notation simplification, we will represent the type of buyer \( \ell \in N_1 \cup N_2 \) as \((a_\ell, d_\ell, v_\ell)\) when we analyze selfish behaviors. However, the heterogeneity on service length will not be ignored in the spectrum allocation.

Without loss of generality, for any buyer \( \ell \in N_1 \cup N_2 \), given a probability \( \mu \) of being served within its active period \([a_\ell, d_\ell]\), the gain of the buyer, i.e., \( G(type|\text{winning probability}) \), can be expressed as

\[
G((a_\ell, d_\ell, v_\ell) | \mu) = v_\ell \cdot L_\ell \cdot \mu. \tag{1}
\]

As explained in the Introduction, it is practical that the information of buyer valuations towards spectrum is available to the auctioneer and may vary with the time. Thus, we define that the distribution of secondary spectrum buyers’ unit valuation with starting time \( t \) follows a probability mass function (PMF) \( f_\ell(t) \) and a corresponding cumulative distribution function \( F_\ell(t) \). Note that in practice, these distributions may be estimated from history. In other words, this time-dependent valuation information is an a priori knowledge for the PO before the auction starts. Similar settings have been considered in [21], [22], and the design of the estimation method is beyond the scope of this paper.

### C. Problem Formulation

In this subsection, we formulate the auction problem which aims to maximize the expected auction revenue of the PO while following the direct revelation principle.

Define \( r_1^1 \) and \( r_2^1 \) as sets of Class I-ISU and Class II-ISU services with demanded starting times equal to \( t \), respectively. Let \( m^\ell \) represent the number of unused channels remaining at the end of \( t \). We further define \( h^\ell \) which includes all service requests up to \( t \), i.e., \( \bigcup_{\ell \in [0, t]} r_1^\ell \cup r_2^\ell \), and all allocation decisions up to \( t - 1 \). An auction mechanism can be described as a pair of allocation and payment rules \((X, P)\), where \( X = \{X(a_\ell, d_\ell, v_\ell)\}_{\ell \in N_1 \cup N_2} \) and \( P = \{P(a_\ell, d_\ell, v_\ell)\}_{\ell \in N_1 \cup N_2} \). Here, \( X(a_\ell, d_\ell, v_\ell) \) indicates the probability that buyer \( \ell \) gets served within \([a_\ell, d_\ell]\), and \( P(a_\ell, d_\ell, v_\ell) \) denotes the expected unit payment of buyer \( \ell \). Thus, the expected total payment of buyer \( \ell \), \( \bar{P}(a_\ell, d_\ell, v_\ell) = L_\ell \cdot P(a_\ell, d_\ell, v_\ell) \). Notice that \( P(a_\ell, d_\ell, v_\ell) \) is also a function of \( X(a_\ell, d_\ell, v_\ell) \). We further define \( x^f(a_\ell, d_\ell, v_\ell) h^\ell \) = 1 means that buyer \( \ell \) can be served in time slot \( t \) given \( h^\ell \), and \( x^f(a_\ell, d_\ell, v_\ell) h^\ell \) = 0, otherwise.

Now, we build constraints of the auction problem as follows:

- **Constraint 1 (Incentive Compatibility):** No buyer can improve its utility by misreporting its type, i.e., for any buyer \( \ell \) with type \((a_\ell, d_\ell, v_\ell)\) but misreport \((a_\ell', d_\ell', v_\ell')\), we have

\[
G((a_\ell, d_\ell, v_\ell)|X(a_\ell, d_\ell, v_\ell)) - \bar{P}(a_\ell, d_\ell, v_\ell) \geq G((a_\ell, d_\ell, v_\ell)|X(a_\ell', d_\ell', v_\ell')) - \bar{P}(a_\ell', d_\ell', v_\ell'), \tag{2}
\]

where \( a_\ell' \), \( d_\ell' \) and \( v_\ell' \) may or may not be equal to \( a_\ell \), \( d_\ell \) and \( v_\ell \), respectively.

- **Constraint 2 (Individual Rationality):** No buyer will be charged more than its gain, i.e.,

\[
G((a_\ell, d_\ell, v_\ell)|X(a_\ell, d_\ell, v_\ell)) - \bar{P}(a_\ell, d_\ell, v_\ell) \geq 0, \tag{3}
\]
• **Constraint 3 (Allocation Feasibility):** Given \( h^t \) for any \( t \in T \), at most \( m^t+1 \) channels can be allocated to secondary spectrum buyers in time slot \( t \). Apparently, the spectrum usage in time slot \( t \) consists of usages from both Class I-SU services and Class II-SU services, called \( \text{Usage}_{\text{class I}}^t \) and \( \text{Usage}_{\text{class II}}^t \), respectively. Since each granted Class I-SU service will only occupy the allocated channel for one time slot, \( \text{Usage}_{\text{class I}}^t \) can be calculated by summing the allocation decisions at time slot \( t \) for all Class I-SU services with their demanded starting times earlier than \( t \), i.e.,

\[
\text{Usage}_{\text{class I}}^t = \sum_{\tau \leq t} \sum_{r \in \mathcal{r}_\tau} x^t(r| h^t),
\]  
where \( x^t(r| h^t) \). Note that, for any Class I-SU service which has been granted before \( t \), its allocation decision at \( t \) equals 0. On the other hand, granted Class II-SU services will not release their occupied channels till the end of their service lengths. Thus, the Class II-SU spectrum usage in time slot \( t \) includes both spectrum usage by newly granted Class II-SU services at \( t \), and the unreleased spectrum usage granted for Class II-SU services starting before \( t \) and ending after \( t \), i.e.,

\[
\text{Usage}_{\text{class II}}^t = \sum_{r \in \mathcal{r}_x} x^t(r| h^t) + \sum_{\tau < t} \sum_{r \in \mathcal{r}_\tau} x^t(r| h^t)
\]

\[
= \sum_{\tau < t} \sum_{r \in \mathcal{r}_\tau} x^t(r| h^t).
\]

The constraint of allocation feasibility can be expressed as \( \text{Usage}_{\text{class I}}^t + \text{Usage}_{\text{class II}}^t \leq m^t \), \( \forall t \in T \). Thus, from (4) and (5), we have

\[
\sum_{\tau \leq t} \sum_{r \in \mathcal{r}_\tau} x^t(r| h^t) \leq m^t.
\]  

From the view of the PO, the expected payment of each buyer with type \((a, d, v)\) who starts its request at \( t \) can be calculated as

\[
f_t(v)P(a, d, v),
\]

where \( f_t(v) \) denotes the PMF of \( v \) on support \( V \) given the service starting time \( t \).

In summary, we can formulate our auction design problem that aims to maximize the expected revenue of the PO over the whole auction frame \( T \) as

**Problem 1:**

\[
\text{Problem 1: } \max_{X, P} \sum_{i=0}^{T-1} \sum_{v \in V} \sum_{t \in T^t} f_t(v)P(a, d, v)\]

\[
s.t. \quad (2), (3), (6),
\]

where \( n^t \) denotes the set of service requests with starting time as \( t \). Note that **Problem 1** is much more difficult than conventional spectrum auction problems [14]–[19]. The challenges are: i) \( P \) is a set of undetermined pricing functions rather than a simple decision vector, and it should be designed to resist both value and time cheating from secondary spectrum buyers; ii) \( X \) and \( P \) have to be solved by jointly considering received bids and the statistics of valuation information; and

• iii) The spectrum allocation for multiple channels within a whole auction frame \( T \) is considerably flexible since buyers have different delay tolerances and service lengths, and thus spectrum allocations for different time slots become correlated and channels can be reused in time by multiple non-overlapped spectrum requests. These flexibilities will definitely make the spectrum allocation much more complicated. To address these issues, in the next section, we first study the characteristics of the pricing function, and then transform the original auction problem to a simpler form with the derivation of the optimal payment rule.

The time-line of our proposed auction framework can be summarized as follows:

• The PO observes the time-dependent valuation information from history before the start of auction frame \( T \).

• At the beginning of \( T \), each SU submits its bid including its time preferences and the price it is willing to pay. Note that, the proposed auction mechanism with direct revelation principle can guarantee each SU to bid truthfully by revealing its type.

• The PO determines a single-step spectrum assignment \( X \) and payments \( P \) which lead to the maximum expected auction revenue over the whole auction frame \( T \) based on its received bids and the known valuation information.

### IV. Optimal Spectrum Auction Mechanism

In this section, the optimal payment rule is first analyzed and derived. Then, the original auction problem (i.e., **Problem 1**) is transformed to a dynamic knapsack problem. After that an effective spectrum allocation algorithm is proposed.

#### A. Problem Transformation

Since all spectrum buyers are risk neutral and have no delay discounting on their valuations, each of them is indifferent to the shift of the winning instant within its active period if the winning probability over its entire active period remains the same. However, as intelligent and selfish players, secondary spectrum buyers may choose to misreport their types if they can benefit from such behaviors. For any spectrum request \( i \in \mathcal{N}_1 \) with misreport \((a_i', d_i', v_i')\), we assume that \( a_i' \geq a_i \) and \( d_i' \leq d_i \), i.e., no Class I-SU request would misreport either an earlier request starting time or a later deadline. That is because reporting \( a_i' < a_i \) or \( d_i' > d_i \) may probably lead to a service out of the preferred range \([a_i, d_i]\), which is not expected by the buyer [17], [18]. For any spectrum request \( j \in \mathcal{N}_2 \) with misreport \((a_j', v_j')\), since \( a_j' \) would be exactly the time for the request \( j \) to obtain the service if it is admitted, reporting a later starting time, i.e., \( a_j' > a_j \), will result in an incomplete service, while reporting \( a_j' < a_j \) may lead to a non-positive utility due to the extra payment for redundant service from \( a_j' \) to \( a_j \). Therefore, for Class II-SU service requests, they may cheat only in valuations, but not in service starting time.

If we generalize the potential misreport of buyer \( j \in \mathcal{N}_2 \) to a 3-tuple as \((a_j', d_j', v_j')\) with \( a_j' = d_j' \), the condition of no time misreport is equivalent to \( a_j' \geq a_j \) and \( d_j' \leq d_j \). Hence, the relevant incentive compatibility constraints of any buyer
\( \ell \in \mathcal{N}_1 \cap \mathcal{N}_2 \) can be represented in a general form by expanding all possibilities as

i) Misreporting the valuation (\( v'_\ell \neq v_\ell \)):

\[
\mathcal{G}((a_\ell, d_\ell, v_\ell)|X(a_\ell, d_\ell, v_\ell)) - \bar{P}(a_\ell, d_\ell, v_\ell) \geq \mathcal{G}((a_\ell, d_\ell, v'_\ell)|X(a_\ell, d_\ell, v'_\ell)).
\]

(8)

ii) Misreporting the request presence (\( a'_\ell \neq a_\ell \) or \( d'_\ell \neq d_\ell \)):

\[
\mathcal{G}((a_\ell, d_\ell, v_\ell)|X(a_\ell, d_\ell, v_\ell)) - \bar{P}(a_\ell, d_\ell, v_\ell) \geq \mathcal{G}((a_\ell, d_\ell, v'_\ell)|X(a'_\ell, d'_\ell, v'_\ell)) - P(a'_\ell, d'_\ell, v'_\ell).
\]

(9)

iii) Any combination of i) and ii):

\[
\mathcal{G}((a_\ell, d_\ell, v_\ell)|X(a_\ell, d_\ell, v_\ell)) - \bar{P}(a_\ell, d_\ell, v_\ell) \geq \mathcal{G}((a_\ell, d_\ell, v'_\ell)|X(a'_\ell, d'_\ell, v'_\ell)) - P(a'_\ell, d'_\ell, v'_\ell).
\]

(10)

where \( a'_\ell, d'_\ell \) and \( v'_\ell \) may or may not be equal to \( a_\ell, d_\ell \) and \( v_\ell \), respectively.

In fact, the incentive compatibility constraint (11) can be relaxed due to the following lemma.

**Lemma 1:** If the incentive compatibility constraints for misreporting valuation (8) and underreporting request presence (9), (10) are satisfied, the constraint for the combination of them (11) will be satisfied automatically.

**Proof:** For a buyer with type \((a_\ell, d_\ell, v_\ell)\) and a time-cheated misreport \((a'_\ell, d'_\ell, v_\ell)\), if the incentive compatibility for underreporting request presence is satisfied, we have

\[
\mathcal{G}((a_\ell, d_\ell, v_\ell)|X(a_\ell, d_\ell, v_\ell)) - \bar{P}(a_\ell, d_\ell, v_\ell) \geq \mathcal{G}((a_\ell, d_\ell, v_\ell)|X(a'_\ell, d'_\ell, v'_\ell)) - P(a'_\ell, d'_\ell, v'_\ell).
\]

(12)

For another buyer with type \((a'_\ell, d'_\ell, v'_\ell)\) and a value-cheated misreport \((a'_\ell, d'_\ell, v'_\ell)\), if the incentive compatibility for misreporting valuation is satisfied, we have

\[
\mathcal{G}((a'_\ell, d'_\ell, v'_\ell)|X(a'_\ell, d'_\ell, v'_\ell)) - \bar{P}(a'_\ell, d'_\ell, v'_\ell) \geq \mathcal{G}((a'_\ell, d'_\ell, v'_\ell)|X(a'_\ell, d'_\ell, v'_\ell)) - P(a'_\ell, d'_\ell, v'_\ell).
\]

(13)

Adding (12) and (13) together yields

\[
P(a_\ell, d_\ell, v_\ell) - \bar{P}(a'_\ell, d'_\ell, v'_\ell)
\]

\[
\leq \mathcal{G}((a_\ell, d_\ell, v_\ell)|X(a_\ell, d_\ell, v_\ell)) + \mathcal{G}((a'_\ell, d'_\ell, v'_\ell)|X(a'_\ell, d'_\ell, v'_\ell)) - \mathcal{G}((a_\ell, d_\ell, v_\ell)|X(a'_\ell, d'_\ell, v'_\ell)) - \mathcal{G}((a'_\ell, d'_\ell, v'_\ell)|X(a'_\ell, d'_\ell, v'_\ell)).
\]

(14)

From the definition in (1), we notice that the gain of a buyer only depends on its valuation and allocation probability. Thus, the following two equations hold.

\[
v_\ell \cdot \mathcal{L}_\ell \cdot X(a'_\ell, d'_\ell, v'_\ell) = \mathcal{G}((a'_\ell, d'_\ell, v'_\ell)|X(a'_\ell, d'_\ell, v'_\ell)) = \mathcal{G}((a_\ell, d_\ell, v_\ell)|X(a_\ell, d_\ell, v_\ell)),
\]

(15)

\[
v_\ell \cdot \mathcal{L}_\ell \cdot X(a'_\ell, d'_\ell, v'_\ell) = \mathcal{G}((a'_\ell, d'_\ell, v'_\ell)|X(a'_\ell, d'_\ell, v'_\ell)) = \mathcal{G}((a_\ell, d_\ell, v_\ell)|X(a_\ell, d_\ell, v_\ell)).
\]

(16)

Substituting (15) and (16) into (14), we can obtain inequality (11).

Therefore, Problem 1 can be reformulated as

**Problem 2:**

\[
\begin{align*}
\text{arg max} & \quad \sum_{t=0}^{T-1} \sum_{\ell \in \mathcal{N}_t} \sum_{v_\ell \in \mathcal{V}} f_t(v_\ell) \bar{P}(a_\ell, d_\ell, v_\ell) \\
\text{s.t.} & \quad (3), (6), (8) - (10).
\end{align*}
\]

Clearly, one key to solve this problem relies on the determination of \( \mathcal{P} \). In order to obtain the optimal payment rule, we can first explore the upper and lower bounds of payment functions from the constraints of incentive compatibility.

**Theorem 1:** Consider an allocation rule \( X \) with a corresponding payment rule \( \mathcal{P} \) such that the auction mechanism \((X, \mathcal{P})\) is incentive-compatible. Then, for any buyer with type \((a_\ell, d_\ell, v_\ell)\), its unit payment \( P(a_\ell, d_\ell, v_\ell) \) (abbreviated to \( P_\ell \)) must satisfy the following two conditions.

\[
P_\ell \leq v_\ell X(a_\ell, d_\ell, v_\ell) - \sigma \sum_{k=1}^{k_t-1} X(a_\ell, d_\ell, k\sigma) - U_\sigma(a_\ell, d_\ell), \quad (17)
\]

\[
P_\ell \geq (v_\ell - \sigma) X(a_\ell, d_\ell, v_\ell) - \sigma \sum_{k=2}^{k_t-1} X(a_\ell, d_\ell, k\sigma) - U_\sigma(a_\ell, d_\ell), \quad (18)
\]

where \( v_\ell = k_t \sigma \) and \( U_\sigma(a_\ell, d_\ell) \) denotes the utility of the buyer with type \((a_\ell, d_\ell, \sigma, 1)\), i.e.,

\[
U_\sigma(a_\ell, d_\ell) = \sigma X(a_\ell, d_\ell, \sigma) - P(a_\ell, d_\ell, \sigma). \quad (19)
\]

**Proof:** See Appendix A.

Comparing the upper bound (17) with the lower bound (18), we have following observations.

**Lemma 2:** For an incentive-compatible mechanism \((X, \mathcal{P})\) such that each \( P_\ell \) satisfies the conditions of both upper and lower bounds in (17) and (18),

- the value of \( P(a_\ell, d_\ell, \sigma) \) is not restricted by bounds;
- the difference between the upper and lower bounds of \( P_\ell \) is less than \( \sigma \).

**Proof:** Consider \( v_\ell = \sigma \). The upper and lower bound conditions of (17) and (18) give

\[
P(a_\ell, d_\ell, \sigma) - \sigma X(a_\ell, d_\ell, \sigma) \leq P(a_\ell, d_\ell, \sigma) \leq P(a_\ell, d_\ell, \sigma).
\]

Apparently, the above inequality always holds, i.e., the incentive compatibility does not impose any restriction on the determination of \( P(a_\ell, d_\ell, \sigma) \).

Furthermore, the difference between right-hand sides of (17) and (18) equals \( \sigma (X(a_\ell, d_\ell, k_t \sigma) - X(a_\ell, d_\ell, \sigma)) \). Since \( X(\cdot) \) refers to a probability which is definitely in the range of \([0, 1]\), such difference must be less than \( \sigma \).

Taking into account Lemma 2, Theorem 1 and the constraint of individual rationality, we can set \( P(a, d, \sigma) = 0 \) and \( X(a, d, \sigma) = 0 \), and then let the unit payment of each buyer \( \ell \) with type \((a_\ell, d_\ell, k_t \sigma)\) where \( k_t \neq 1 \) equals the upper bound defined in (17), i.e.,

**Payment rule:**

\[
P(a_\ell, d_\ell, v_\ell) = k_t \sigma X(a_\ell, d_\ell, k_t \sigma) - \sigma \sum_{k=1}^{k_t-1} X(a_\ell, d_\ell, k\sigma), \quad (20)
\]

if \( X(a_\ell, d_\ell, k_t \sigma) \neq 0 \); and \( P(a_\ell, d_\ell, v_\ell) = 0 \), otherwise.
In Section V, we will show that our proposed auction mechanism with such payment rule can satisfy the direct revelation principle. By substituting (20) into Problem 2, constraints (3), (6), (8)-(10) can be further simplified according to the following proposition.

**Proposition 1:** Applying the payment rule designed in (20) for each buyer \( \ell \in N_1 \cup N_2 \), the constraints of direct revelation principle can be simplified as

1. **C1:** Resisting misreports of later starting time,
   \[
   \sum_{\kappa=1}^{\kappa_{\ell}-1} X(a_{\ell}, d_{\ell}, \kappa \sigma) \geq \sum_{\kappa=1}^{\kappa_{\ell}-1} X(a'_{\ell}, d_{\ell}, \kappa \sigma), \text{ if } a_{\ell} \leq a'_{\ell}. \tag{21}
   \]
2. **C2:** Resisting misreports of earlier deadline,
   \[
   \sum_{\kappa=1}^{\kappa_{\ell}-1} X(a_{\ell}, d_{\ell}, \kappa \sigma) \geq \sum_{\kappa=1}^{\kappa_{\ell}-1} X(a'_{\ell}, d_{\ell}, \kappa \sigma), \text{ if } d_{\ell} \geq d'_{\ell}. \tag{22}
   \]
3. **C3:** Valuation monotonicity,
   \[
   X(a, d, v) \geq X(a, d, \bar{v}), \forall \bar{v} \geq v, \forall a, d \in T. \tag{23}
   \]
4. **C4:** Allocation feasibility constraint as in (6), \( \forall t \in T. \)

**Proof:** See Appendix B.

Moreover, the objective function of Problem 2 can also be transformed by adopting the payment rule (20) as shown in Proposition 2.

**Proposition 2:** By substituting (20) to (7), the objective function of Problem 2 can be reformulated as

\[
\arg \max_{\mathbf{X}} \sum_{t=0}^{T-1} \sum_{v \in V} f_t(v) \left( \sum_{\ell \in n^t} X(a_{\ell}, d_{\ell}, v_t) \eta(a_{\ell}, d_{\ell}, v_t) \mathcal{L}_t \right), \tag{24}
\]

where

\[
\eta(a_{\ell}, d_{\ell}, v_t) = v_t - \frac{1}{\sigma} - F_{\alpha_{\ell}}(v_t). \tag{25}
\]

Since \( \eta(a_{\ell}, d_{\ell}, v_t) \) is a function of the truthful value and the time-dependent valuation distribution only, we name it as the **unit virtual valuation** for spectrum request \( (a_{\ell}, d_{\ell}, v_t, \mathcal{L}_t) \) in the following.

**Proof:** Consider the expected revenue of the PO in any time slot \( t \in T \). By adding and subtracting an identical term, we have

\[
\sum_{v \in V} \sum_{\ell \in n^t} f_t(v_t) P(a_{\ell}, d_{\ell}, v_t) \\
= \sum_{v \in V} \sum_{\ell \in n^t} f_t(v_t) X(a_{\ell}, d_{\ell}, v_t) v_t \mathcal{L}_t \\
+ \sum_{v \in V} \sum_{\ell \in n^t} f_t(v_t) (P(a_{\ell}, d_{\ell}, v_t) - X(a_{\ell}, d_{\ell}, v_t) v_t \mathcal{L}_t).
\]

With (44) in Appendix B, we obtain that

\[
\sum_{v \in V} f_t(v_t) (P(a_{\ell}, d_{\ell}, v_t) - X(a_{\ell}, d_{\ell}, v_t) v_t \mathcal{L}_t) \\
= -\mathcal{L}_t \sigma \sum_{v \in V} f_t(v_t) \sum_{\kappa=1}^{\kappa_{\ell}} X(a_{\ell}, d_{\ell}, \kappa \sigma) \frac{f_t(k \sigma)}{f_t(k \sigma)} \\
= -\mathcal{L}_t \sigma \sum_{v \in V} \left( 1 - \frac{F_{\xi_{\ell}}(v_t)}{f_t(v_t)} \right) X(a_{\ell}, d_{\ell}, v_t) f_t(v_t). \tag{27}
\]

Substituting (27) back into (26) results in

\[
\sum_{\ell \in n^t} \sum_{v \in V} f_t(v_t) P(a_{\ell}, d_{\ell}, v_t) \\
= \sum_{v \in V} \sum_{\ell \in n^t} f_t(v) X(a_{\ell}, d_{\ell}, v_t) \left( v_t - \frac{1}{\sigma} - F_{\alpha_{\ell}}(v_t) \right) \mathcal{L}_t. \tag{28}
\]

Therefore, Proposition 2 is proved.

Based on Propositions 1 and 2, the proposed auction problem can now be rewritten as

**Problem 3:**

\[
\arg \max_{\mathbf{X}} \sum_{t=0}^{T-1} \sum_{v \in V} f_t(v) \left( \sum_{\ell \in n^t} X(a_{\ell}, d_{\ell}, v_t) \eta(a_{\ell}, d_{\ell}, v_t) \mathcal{L}_t \right) \tag{29}
\]

subject to **C1 - C4**.

Note that, different from the static single-item stochastic auction [34], where the buyer with the highest virtual valuation is the only winner, the allocation problem defined in Problem 3 is much more complicated due to the heterogeneous time requirements of SUs and multiple available channels from the PO. In the following, an effective channel allocation algorithm is proposed.

**B. Spectrum Allocation Algorithm**

Since the PO receives all bidding information at the beginning of each frame, the goal of the proposed spectrum allocation problem can be simply expressed as deciding \( X(a_{\ell}, d_{\ell}, v_t, \mathcal{L}_t) \) for each buyer \( \ell \in N_1 \cup N_2 \) so as to maximize the total virtual valuation of winners. We first relax the constraints in Problem 3 by ignoring **C1 - C3** and focus on designing a feasible allocation which leads to the maximum auction revenue. In Section V, we can prove that constraints **C1 - C3** are satisfied automatically. By ignoring **C1 - C3**, the spectrum allocation problem becomes

**Problem 4:**

\[
\arg \max_{\mathbf{X}} \sum_{\ell \in N_1 \cup N_2} X(a_{\ell}, d_{\ell}, v_t, \mathcal{L}_t) \eta(a_{\ell}, d_{\ell}, v_t) \mathcal{L}_t \tag{30}
\]

subject to **C4**.

Apparently, Problem 4 is a dynamic knapsack problem with dynamic presence of items (buyers), and thus it is NP-hard [35]. Moreover, unlike traditional dynamic knapsack problems [25], [26] where items allocated to the knapsack will remain till the end of the time frame, secondary spectrum buyers in Problem 4 have heterogeneous service lengths, and granted buyers will release their occupied channels at the end of their required service times. Time variant allocation problems are commonly solved by the forward or backward inductions. The forward induction allocates all remaining unoccupied channels in each time slot by solving a static knapsack problem with regard to all active (i.e., arrived and not expired) but unallocated spectrum buyers, and decisions are made from the beginning of the time frame (time slot 0) to the end (time slot \( T - 1 \)). While, the backward induction starts the channel allocation from the end of the time frame (time slot \( T - 1 \)) back to the beginning (time slot 0), and all previous decisions (i.e., for future time slots) keep updating with the decision of the current time slot until the channel allocation for time slot.
has been optimized. However, neither of these two methods can be employed for Problem 4.

Specifically, if forward induction is applied for spectrum assignment from time slot 0 to \( T - 1 \), since granting a Class II-SU request \( \ell \) at \( t \) on a certain channel will block access opportunities to this channel during \([t, t + L_\ell]\), allocation decisions for Class II-SU services at any \( t \) may decrease chances of accepting future higher-payment buyers after \( t \). Similarly, if backward induction is adopted for spectrum assignment from time slot \( T - 1 \) back to 0, granting a Class II-SU service request \( \ell \) at \( t \) may preempt some already allocated buyers so that the channel assignment becomes suboptimal. Fig. 2 shows an example of spectrum allocation and demonstrates deficits of using forward and backward inductions. In this example, we consider an allocation among 5 Class I-SU and 2 Class II-SU service requests in the scenario with 2 channels and 4 time slots. The time requirement and the virtual valuation of each buyer are depicted in Fig. 2(a). The detailed procedure of the grouping method is shown in Algorithm 1. Let \( G \) include the indices of buyers in \( \mathcal{O} \). The following grouping procedure will follow this order.

For each grouping iteration, we first select the remaining unallocated buyer with the lowest index, and then keep adding the next unallocated buyer in order, which does not conflict with buyers who are already in the group. The grouping iteration stops when either \( T \) time slots have been completely occupied or no more non-conflict unallocated buyer can be accommodated in this group.

The detailed procedure of the grouping method is shown in Algorithm 1. Let \( G \) include the indices of buyers in the group, and set \( \mathcal{O} \) include all time slots that are already occupied. \( \mathcal{S}_t \) and \( \mathcal{S}_p \) are sets of time slots for potential access by Class I-SU services and times slots occupying by Class II-SU requests in the group, respectively. Furthermore, \( N^p \) (\( N^p_2 \)) indicates the number of Class I-SU (Class II-SU) services in the group. In addition, we define \( |\cdot| \) as the size of a set, and \( A \setminus B \) as the complement set of \( B \) in \( A \).

The set \( \mathcal{O} \) consists of two parts: 1) the slots which lose their allocation flexibility because of Class I-SU services in the group (e.g., slots \( \{t, t+1, t+2\} \) will lose their allocation flexibility if there are 3 grouped Class I-SU services all with the same \( \mathcal{I} = \{t, t+1, t+2\} \); 2) the slots occupied by Class

\[
\eta_1 \geq \eta_2 \geq \ldots \geq \eta_\ell, \ldots \geq \eta_N, \tag{29}
\]

where \( N \) is the total number of buyers (i.e., the size of set \( \mathcal{N}_1 \cup \mathcal{N}_2 \)). Note that the indices of buyers have been rearranged and the following grouping procedure will follow this order.

As indicated previously, it does not matter when the buyer is going to be granted, but whether a buyer can be granted within its active period. Hence, we can group non-conflict high-payment buyers together until all \( T \) time slots of one channel is totally allocated. Since the PO owns \( M \) homogeneous orthogonal channels to auction, \( M \) buyer groups will be generated and all grouped buyers become winners in the auction. The detailed solution procedure is as follows. Initially, let \( A_\ell \in \{0, 1\} \) indicate whether buyer \( \ell \in \mathcal{N}_1 \cup \mathcal{N}_2 \) is granted, and \( \text{Cate}_\ell \in \{0, 1\} \) represent the category that \( \ell \) belongs to (Class I or Class II). For \( \ell \in \mathcal{N}_1 \), buyer \( \ell \) only demands one of the slots from its active period \( \mathcal{I}_\ell = \{a_\ell, a_\ell + 1, \ldots, d_\ell\} \), while for \( \ell \in \mathcal{N}_2 \), \( \mathcal{I}_\ell \) is the set of consecutive time slots that \( \ell \) requested, i.e., \( \mathcal{I}_\ell = \{a_\ell, a_\ell + 1, \ldots, a_\ell + L_\ell\} \). In order to balance the weight values of Class I-SU and Class II-SU service requests, we propose two buyer grouping methods which are based on unit virtual valuation (i.e., \( \eta_\ell, \forall \ell \in \mathcal{N}_1 \cup \mathcal{N}_2 \)) and service virtual valuation (i.e., \( \eta_\ell \cdot L_\ell, \forall \ell \in \mathcal{N}_1 \cup \mathcal{N}_2 \)), respectively.

**Buyer Grouping based on Unit Virtual Valuation:** After receiving all bids, the PO calculates \( \eta_\ell, \forall \ell \in \mathcal{N}_1 \cup \mathcal{N}_2 \), according to (25), and then sorts all buyers based on a decreasing order of their unit virtual valuations as

\[
\eta_1 \geq \eta_2 \geq \ldots \geq \eta_\ell, \ldots \geq \eta_N.
\]

\[
\text{Cate}_\ell \in \{0, 1\} \tag{29}
\]

\[
\eta_1 \geq \eta_2 \geq \ldots \geq \eta_\ell, \ldots \geq \eta_N,
\]
II-SU services in the group. With non-conflict requirement for grouping new buyers, we can set the grouping criteria for Class I-SU service request $\ell$ as 1) the active period of $\ell$ is not a subset of $O$, i.e., $I_{\ell} \nsubseteq O$; 2) after grouping $\ell$, the number of Class I-SU services in the group is not larger than $|S_{\ell} \cup I_{\ell} \cap (T \setminus S_{\ell})|$. For Class II-SU service request $\ell$, the grouping criteria are 1) the time requirement of $\ell$ is not overlapped with $O$; 2) grouping $\ell$ will not interfere the grouping criteria for Class I-SU services that are already in the group. As an example, consider a buyer group which has 3 Class I-SU services with $I_{\ell}^{C1} = \{1, 2\}$, $I_{\ell}^{C2} = \{1, 2\}$, $I_{\ell}^{C3} = \{2, 3, 4, 5, 6\}$, and 1 Class II-SU service with $I_{\ell}^{C2} = \{6, 7\}$. Apparently, the set of occupied slots $O = \{1, 2, 6, 7\}$, since slots 1 and 2 are filled with two Class I-SU services (i.e., $I_{\ell}^{C1}$ and $I_{\ell}^{C2}$), and slots 6 and 7 are allocated to $I_{\ell}^{C2}$. Now, we can check the grouping criteria for other Class I-SU service requests with $I_{\ell}^{C1} = \{1, 2\}$, $I_{\ell}^{C2} = \{3, 4\}$, $I_{\ell}^{C6} = \{4, 5\}$, $I_{\ell}^{C7} = \{3, 4\}$. Apparently, $I_{\ell}^{C4}$ cannot be grouped due to the first criterion. In addition, if $I_{\ell}^{C1}$ and $I_{\ell}^{C6}$ have been grouped, $I_{\ell}^{C3}$ can no longer be grouped, because grouping $\ell$ will make the number of available time slots for these requests $|S_{\ell} \cup I_{\ell} \cap (T \setminus S_{\ell})| = 5$, which is less than the number of total Class I-SU services ($N_{\ell} = 6$). We can also check the grouping criteria for other Class II-SU service requests with $I_{\ell}^{C2} = \{2, 3, 4\}$, $I_{\ell}^{C5} = \{3, 4, 5\}$, $I_{\ell}^{C4} = \{3, 4\}$. With the first criterion, $I_{\ell}^{C2}$ is eliminated since it overlaps with $O$. Furthermore, $I_{\ell}^{C7}$ does not satisfy the second criterion since grouping it will preempt the allocation of $I_{\ell}^{C1}$. Therefore, only $I_{\ell}^{C4} = \{3, 4\}$ can be grouped.

**Buyer Grouping based on Service Virtual Valuation:** Similar as in the commodity market, the seller does not only prefer buyers with higher unit payments, but also buyers with higher payments for large bundles of goods. The reason is that selling a bundle of items together rather than a single item separately can reduce the risk in future market. Thus, after receiving all the bids, the PO sorts buyers based on a decreasing order of their service virtual valuations as

$$\eta_{1}L_{1} \geq \eta_{2}L_{2} \geq \ldots \geq \eta_{L}L_{L} \geq \eta_{N}L_{N}, \quad (30)$$

Similarly, the indices of buyers are rearranged and such order will be used in buyer grouping procedure. Since the procedure of buyer grouping in this case is the same as that based on unit virtual valuation except that buyers indices are different, Algorithm 1 is also applicable here.

**Spectrum Allocation based on Buyer Groupings:** Starting from channel 1, two different buyer groups can be generated according to unit and service virtual valuation, respectively. The buyer group with higher total virtual valuation (i.e., $\sum \eta_{L}$) will be allocated. Meanwhile, all buyers in this group will be marked as allocated and will not be considered in future buyer grouping. The procedure continues until all $M$ channels have been occupied or all buyers have been allocated. The detailed steps of the proposed algorithm is presented in Algorithm 2.

Consider the same example shown in Fig. 2. First of all, we can sort seven buyers as $B_{3}, B_{5}, B_{2}, B_{1}, B_{7}, B_{1}, B_{6}$ and $B_{2}, B_{1}, B_{3}, B_{5}, B_{4}, B_{7}, B_{6}$ based on (29) and (30), respectively. For the allocation of channel 1, we can generate two buyer groups according to different buyer orders as $G_{1}^{1} = \{B_{3}, B_{5}, B_{1}, B_{7}\}$ and $G_{2}^{1} = \{B_{2}, B_{3}\}$. Since the total virtual valuation for $G_{1}^{1}$ is $\eta_{3} + \eta_{5} + \eta_{1} + \eta_{7} = 34.5$, while for $G_{2}^{1}$ is $3\eta_{2} + \eta_{5} = 34$, $G_{1}^{1}$ will be allocated to channel 1. Next, for the allocation of channel 2, two same buyer groups will be generated as $G_{1}^{2} = G_{2}^{2} = \{B_{2}, B_{6}\}$. Thus, $B_{2}, B_{6}$ will be allocated to channel 2. Apparently, the allocation produced by Algorithm 2 has the same total virtual valuation as the optimal one. In Section VI, more simulation results are provided to illustrate this fact.

**Algorithm 2:** Allocation based on Buyer Groups

**Input:** Number of channels $M$; Number time slots $T$; Time requirements $(a_{t}, d_{t}, \lambda_{t}), \forall t \in N_{p} \cup N_{s}^{O}$; $A, I, C_{\text{rate}}$.

**Output:** Decision of the spectrum Allocation.

```plaintext
1 for Ch = 1 to M do
2 Rearrange buyer indices according to (29)
3 Call Buyer Grouping Algorithm
4 $G_{Ch}^{1} = G, \text{sum}_{1} = 0$
5 for $\ell \in G$ do
6 \[ A_{\ell} = 0, \text{sum}_{1} = \text{sum}_{1} + \eta_{\ell}\lambda_{\ell} \]
7 Rearrange buyers indices according to (30)
8 Call Buyer Grouping Algorithm
9 $G_{Ch}^{2} = G, \text{sum}_{2} = 0$
10 for $\ell \in G$ do
11 \[ A_{\ell} = 0, \text{sum}_{2} = \text{sum}_{2} + \eta_{\ell}\lambda_{\ell} \]
12 if $\text{sum}_{1} \geq \text{sum}_{2}$ then
13 \[ A_{\ell} = 1, \forall \ell \in G_{Ch}^{1} \text{ and return } G_{Ch}^{1} \]
14 else
15 \[ A_{\ell} = 1, \forall \ell \in G_{Ch}^{2} \text{ and return } G_{Ch}^{2} \]
```
This condition implies that when the starting time and the deadline are fixed, the hazard rate is monotonically increased with the valuation [34]. Furthermore, for \([a, d] \subset [\hat{a}, \hat{d}]\), such condition indicates a first order stochastic dominance [36], i.e.,

\[
E[v|(a, d)] \geq E[v|\{\hat{a}, \hat{d}\}],
\]

which means that a shorter active period results in a higher expected valuation. Commonly, wireless users with lower service delay tolerances have more urgent transmission requirements. It is reasonable that impatient spectrum buyers are willing to pay more for fulfilling their requests. In other words, impatient buyers with shorter active periods are expected to have higher valuations than patient buyers with longer active periods. Thus, the MHR condition is meet for a practical CR network.

With the satisfaction of MHR condition, we have \(\frac{\partial \eta(a, d)}{\partial v} \geq 0\). Thus,

\[
\frac{\partial \eta(a, d, v)}{\partial v} = \frac{\partial}{\partial v}(v - 1/\lambda_a(v)) = 1 + \frac{1}{[\lambda_a(v)]^2} \frac{\partial \lambda_a(v)}{\partial v} > 0,
\]

which means that \(\eta(a, d, v)\) is also monotonically increasing with the valuation \(v\). Hence, the unit virtual valuation of type \((a, d, \hat{v})\) is higher than \((a, d, v)\), and so is the service virtual valuation of \((a, d, \hat{v})\) since they have the same service length. According to the designed allocation algorithm, \((a, d, \hat{v})\) is always ranked in front of \((a, d, v)\) in both ordering procedures (29) and (30). As a result, the buyer with \(\hat{v} \geq v\) has a higher priority in grouping and allocation so that the buyer will definitely win with \(\hat{v}\) if it can win with \(v\).

**Lemma 4:** In the proposed spectrum auction, no buyer can improve its utility by misreporting a later service starting time or an earlier deadline.

**Proof:** According to C1 and C2, we need to prove inequalities (21) and (22) are satisfied. Given a valuation \(v\), the allocation probability of the buyer only relies on the probability of whether it could be grouped in Algorithm 1. Intuitively, the buyer should report its active period as wider as possible in order to avoid conflict with other higher-payment buyers in the grouping process so that its allocation probability can be increased. Thus, we have

\[
X(a, d, v) \geq X(a', d, v), \quad \forall a \leq a'. \tag{35}
\]

\[
X(a, d, v) \geq X(a, d', v), \quad \forall d \geq d'. \tag{36}
\]

With simple manipulations, (35) and (36) directly indicate (21) and (22).

**Theorem 2:** The proposed auction mechanism is incentive-compatible, i.e., all secondary wireless users will behave truthfully in the spectrum auction.

**Proof:** This can be concluded by Lemmas 3 and 4.

**VI. SIMULATION RESULTS**

In this section, simulations are conducted to evaluate the proposed spectrum auction mechanism. With the consideration of differential secondary service provisioning and time-dependent buyer stochastic valuation, the superiority of our designed spectrum allocation algorithm and its performance in terms of auction revenue are presented.

---

**Fig. 3.** Buyer unit valuation with different starting times.

**A. Simulation Settings**

Consider a CR network with one PO who runs a spectrum auction over the frame \(\mathcal{T}\) which is set as an hour. The auction frame is further divided into 6 time slots where each lasts for 10 minutes. We assume that the number of auctioned channels, i.e., \(M\), for secondary services at the beginning of \(\mathcal{T}\) is 24. Thus, the total number of time-frequency chunks is 144. Same settings have been employed in [15]. For simulation purpose, we consider an overloaded scenario by letting the number of Class I-SU services \(N_1 = 400\), and Class II-SU services \(N_2 = 200\). For each Class I secondary buyer \(i\), its service starting time \(a_i\) is randomly selected in \([0, 6]\), while the deadline \(d_i\) is random in \([a_i, 6]\). For each Class II secondary buyer \(j\), \(a_j\) and \(d_j\) are exactly the same, and are random in \([0, 6]\), while its service length \(L_j\) is determined randomly in \([0, 6 - a_j]\). In addition, let \(\sigma = 1\) and \(K = 100\). The buyer unit valuation distribution at any starting time is uniformly distributed, and the mean values follow a bell form over \(\mathcal{T}\) with \(t = 3\) as the axis of symmetry and 0.75 as the variance, as shown in Fig. 3. In the figure, Valuation-mean presents a bell curve corresponding to mean values, and Valuation-max is the maximum valuation which equals the double of Valuation-mean due to the assumption of uniform distributions. Note that, all the rest of simulation results are based on the valuations selected from Fig. 3, and some parameters may vary according to evaluation scenarios.

**B. Numerical Results**

Fig. 4 illustrates the buyer unit virtual valuation given the unit valuation and the distribution as shown in Fig 3. From the figure, we can observe that the trends of curves are opposite to those in Fig 3, which reflects different significance of a fixed unit valuation along the times. For example, consider

---

2As analyzed in Lemma 3, our designed auction mechanism does not rely on the specific distribution of buyer valuation information except the satisfaction of the MHR condition which is naturally satisfied by many commonly used distributions in wireless networks, such as exponential, uniform, and binomial distributions. This implies that similar observations can be obtained by employing different valuation distributions. Since our focus in simulations is to show the applicability and efficiency of our proposed auction mechanism compared to counterparts, we choose the uniform distribution as an illustrative example for evaluation purposes.
two starting times of $t = 1$ and $2$. Given a unit valuation $v = 20$, since $v$ approaches the expected valuation at $t = 1$ (Valuation-mean = 20 from Fig 3), but is much smaller than the expectation at $t = 2$ (Valuation-mean = 38 from Fig 3), the significance of $v = 20$ at $t = 2$ is smaller than that at $t = 1$, which results in a smaller unit virtual valuation at $t = 2$. Note that, since $v = 20$ has already exceeded Valuation-max for $t = 0$ and $6$, the curve of $v = 20$ in the figure does not have values at these two points. Furthermore, Fig. 4 also verifies that, with the same starting time, the buyer which has higher unit valuation will obtain a higher unit virtual valuation, i.e., the MHR condition is satisfied under our settings.

Fig. 5 compares the performance of buyer-grouping based allocation algorithms according to different buyer orders. It shows that buyer grouping based on the unit virtual valuation obtains a higher total virtual valuation from winners than that based on the service virtual valuation. That is because choosing buyers with higher unit virtual valuation as winners commonly leads to a better performance when the number of buyers is much larger than the total number of time-frequency chunks. However, the unit virtual valuation-based allocation may not be always the better choice for each individual channel (or buyer group). The proposed allocation algorithm chooses the better outcome from two different buyer orders for each channel, and thus achieves the best performance. In addition, when the number of channels increases, the gaps among the three curves become larger due to the accumulative effect. However, when the total number of time-frequency chunks is large enough so that most higher-payment buyers can be served regardless of grouping orders, three curves converge.

Fig. 6 illustrates the superiority of the proposed algorithm on spectrum allocation efficiency in terms of the total virtual valuation obtained from winners. The optimal solution is obtained based on the exhaustive searching algorithm, which checks and compares the outcomes of all possible allocations. Due to the non-polynomial time computational complexity, we assume that $\mathcal{N}_1$ ($\mathcal{N}_2$) only consists of 8 (4) different kinds of buyers, where the number of each kind of buyers is 50. By taking the average over 100 runs, we can see in the figure that the proposed algorithm can achieve nearly the same performance as the optimal one, and outperforms both the forward and backward induction-based algorithms. This is because the proposed algorithm does not only optimize the channel allocation over the whole time frame (compared to the forward induction-based algorithm), but also employs buyer groupings (i.e., Algorithm 1) to avoid the impact from any high-valued individual buyer on allocation decisions (compared to the backward induction-based algorithm). Moreover, since the time flexibility of the channel allocation increases significantly with the number of time slots, the performance gaps between our proposed algorithm and others become more obvious when the auction frame gets longer.

Fig. 7 further examines the performance of different spec-
Auction revenue of the proposed mechanism.

Fig. 8. Auction revenue of the proposed mechanism.

Auction revenue of the proposed mechanism.

Fig. 9. Comparison of different auction mechanisms with $M = 24$.

Comparison of different auction mechanisms $N_1 = 400$.

Fig. 10. Comparison of different auction mechanisms $N_1 = 400$.

In this paper, a spectrum auction with time-dependent valuation information for heterogeneous secondary service provisioning in CR networks was discussed. In our framework, each spectrum buyer bids with its time and price preferences, and the PO determines the spectrum allocation based on each spectrum buyer’s bids with its time and price preferences. To eliminate this effect, we compare the auction revenues between two settings, both with 600 buyers (i.e., $N_1 = 400, N_2 = 200$ and $N_1 = 200, N_2 = 400$).

Since the demand of time slots from a Class II secondary buyer is always larger than that from a Class I secondary buyer, we can observe from the figure that the more Class II buyers we have, the higher competition intensity will be produced, and finally a higher auction revenue can be obtained.

In order to demonstrate the improvement on auction revenue by adopting the proposed mechanism, two offline optimal time-frequency based spectrum auction mechanisms, i.e., TDSA [15] and SMASHER-GR [14], and an online spectrum auction mechanism, i.e., Topaz [17], are simulated and compared. TDSA requires all buyers to claim fixed time requirements, while SMASHER-GR and Topaz can allow time-flexible requests. However, none of them utilized the time-dependent buyer valuation information. For a fair comparison, we assume $N_2 = 0$ since these works cannot deal with both continuous and disjointed spectrum requests at the same time.

Fig. 9 compares four auction mechanisms with respect to the number of secondary spectrum buyers. From this figure, we can observe that auction revenues are higher for mechanisms with time flexibilities (i.e., the performance of the proposed mechanism, SMASHER-GR and Topaz are better than that of TDSA). Intuitively, such observation results from the less stringent constraint on resource allocations. Furthermore, SMASHER-GR outperforms Topaz because the information of all requests are taken into account in offline-based allocation algorithms, while online-based algorithms can only myopically optimize spectrum allocations for each individual time slot. Moreover, the proposed mechanism achieves the best performance due to the employment of the time-dependent buyer valuation information. We further compare these four auction mechanisms with respect to the number of channels in Fig. 10, and similar observations as in Fig. 9 can be obtained.

VII. Conclusion

In this paper, a spectrum auction with time-dependent valuation information for heterogeneous secondary service provisioning in CR networks was discussed. In our framework, each spectrum buyer bids with its time and price preferences, and the PO determines the spectrum allocation based on its received bids and the known time-dependent valuation.
information so as to maximize the expected auction revenue. An auction mechanism, which satisfies the direct revelation principle, was proposed. Besides, an effective spectrum allocation algorithm based on buyer grouping method was designed which can handle the temporal flexibility and reusability among different classes of wireless services. Simulation results showed that the proposed auction mechanism can improve spectrum allocation efficiency and auction revenue compared to counterparts.

**APPENDIX A**

**Proof of Theorem 1**

**Proof:** To prove (17), we first consider a buyer with type \((a, d, κσ)\) and a misreport \((a, d, (k_ℓ - 1)σ)\). The incentive compatibility constraint directly implies

\[
k_ℓσX(a, d, κσ) - P(a, d, κσ) ≥ k_ℓσX(a, d, (k_ℓ - 1)σ) - P(a, d, (k_ℓ - 1)σ).
\]  

(37)

After some simple manipulations, (37) can be rewritten as

\[
Pr ≤ P_{ℓ-1} + k_ℓσ[X(a, d, κσ) - X(a, d, (k_ℓ - 1)σ)].
\]  

(38)

where \(P_{ℓ-1}\) stands for \(P(a, d, (k_ℓ - 1)σ)\).

From (38), the incentive compatibility constraint for the type \((a, d, (κ + 1)σ)\) with a misreport \((a, d, κσ)\) implies

\[
P_{κ+1} ≤ P_κ + (κ + 1)σ[X(a, d, (κ + 1)σ) - X(a, d, κσ)].
\]  

(39)

∀κ \in \{1, 2, ..., k_ℓ - 1\}. Take all possible values of κ into (39), and then sum those generated inequalities as

\[
\frac{k_ℓ-1}{2}\sum_{κ=1} P_{κ+1} ≤ \frac{k_ℓ-1}{2}\sum_{κ=1} P_κ + (2κ+1)σX(a, d, κσ) - X(a, d, κσ).
\]  

(40)

From (40), we have

\[
P_{ℓ} ≤ P(a, d, κσ) + k_ℓσX(a, d, κσ) - 2κσX(a, d, κσ) - \frac{1}{2}\sum_{κ=1}^{k_ℓ-1} X(a, d, κσ).
\]  

(41)

Substituting \(U_a(a, d, κ) = σX(a, d, κσ) - P(a, d, κσ)\) into (41) will result in (17).

Similarly, to prove (18), we can consider the incentive compatibility constraint for the type \((a, d, κσ)\) with a misreport \((a, d, (κ + 1)σ)\) as

\[
P_κ ≤ P_{κ+1} + κσ[X(a, d, κσ) - X(a, d, (κ + 1)σ)].
\]  

(42)

Summing the above inequalities with regard to all possible values of κ yields

\[
P(a, d, κσ) ≤ P_{ℓ} - (k_ℓ - 1)σX(a, d, κσ) + σX(a, d, κσ)
\]  

(43)

Substituting \(U_a(a, d, κσ) = σX(a, d, κσ) - P(a, d, κσ)\) into (43) will lead to (18).

**APPENDIX B**

**Proof of Proposition 1**

**Proof:** For any buyer \(ℓ \in N_1 ∪ N_2\) with type \((a, d, κσ)\) where \(k_ℓ ≠ 1\), substituting the payment function (20) to the utility function of buyer \(ℓ\) becomes

\[
U_ℓ = G((a, d, κσ)|X(a, d, κσ)) - P(a, d, κσ) = L_ℓ(κσX(a, d, κσ)) - L_ℓX(a, d, κσ)
\]  

\[
-σ\sum_{κ=1}^{k_ℓ-1} X(a, d, κσ) = L_ℓσ\sum_{κ=1}^{k_ℓ-1} X(a, d, κσ).
\]  

(44)

The above expression is always non-negative since all terms are non-negative. Furthermore, the utility of a buyer is zero when \(k_ℓ = 1\). Therefore, the constraint for individual rationality will be satisfied automatically with the designed payment so that the constraint (3) can be removed. Similarly, substituting (20) to (9) and (10), constraints C1 and C2 can be proved. Besides, the constraint of allocation feasibility C4 remains exactly the same as (6).

Now, the only remaining problem is to prove that the constraint for resisting misreports of valuation (8) can be replaced by the valuation monotonicity C3. Consider two buyer types with \((a, d, v)\) and \((a, d, ̂v)\). Since these two requests have the same temporal requirement, we omit \((a, d)\) for notation simplicity. First, we can assume by the way of contradiction that

\[
X(̂v) < X(v), \text{ if } ̂v ≥ v.
\]  

(45)

Substituting (1) into (8) and dividing \(L\) on both sides of the inequality, we have

\[
vX(v) - vX(v') ≥ P(v) - P(v').
\]  

(46)

Based on our assumption in (45), we have

\[
̂vX(̂v) - ̂vX(̂v') ≥ P(v) - P(̂v).
\]  

(47)

Consider another buyer with its truthful type as \((a, d, v)\) while misreports \((a, d, ̂v)\). Then, according to the constraint of incentive compatibility, we have

\[
̂vX(̂v) - P(̂v) ≥ ̂vX(v) - P(v).
\]  

(48)

Obviously, (47) contradicts (48). Thus, the assumption (45) is false, and in turn C3 holds.

**REFERENCES**


Changyan Yi (S’16) received the B.Sc. degree from Guilin University of Electronic Technology, China, in 2012, and the M.Sc. degree from University of Manitoba, Winnipeg, MB, Canada, in 2014. He is currently working toward the Ph.D. degree in electrical and computer engineering, University of Manitoba. He was awarded Edward R. Toporeck Graduate Fellowship in Engineering in 2014, 2015 (twice), University of Manitoba Graduate Fellowship (UMGF) for 2015-2018, and IEEE ComSoc Student Travel Grant for IEEE Globecom 2016. His research interests include algorithmic game, optimization and queuing theories for radio resource management, prioritized scheduling and network economics in wireless communications.

Jun Cai (M’04-SM’14) received the B.Sc. and M.Sc. degrees from Xi’an Jiaotong University, Xi’an, China, in 1996 and 1999, respectively, and the Ph.D. degree from the University of Waterloo, ON, Canada, in 2004, all in electrical engineering. From June 2004 to April 2006, he was with McMaster University, Hamilton, ON, as a Natural Sciences and Engineering Research Council of Canada Postdoctoral Fellow. Since July 2006, he has been with the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB, Canada, where he is currently an Associate Professor. His current research interests include energy-efficient and green communications, dynamic spectrum management and cognitive radio, radio resource management in wireless communications networks, and performance analysis. Dr. Cai served as the Technical Program Committee Co-Chair for the IEEE Vehicular Technology Conference 2011 Wireless Applications and Services Track, the IEEE Global Communications Conference (Globecom) 2010 Wireless Communications Symposium, and International Wireless Communications and Mobile Computing (IWCMC) Conference 2008 General Symposium; the Publicity Co-Chair for IWCMC in 2010, 2011, 2013, and 2014; and the Registration Chair for the First International Conference on Heterogeneous Networking for Quality, Reliability, Security and Robustness (QShine) in 2005. He also served on the editorial board of the Journal of Computer Systems, Networks, and Communications and as a Guest Editor of the special issue of the Association for Computing Machinery Mobile Networks and Applications. He received the Best Paper Award from Chinacom in 2013, the Rh Award for outstanding contributions to research in applied sciences in 2012 from the University of Manitoba, and the Outstanding Service Award from IEEE Globecom in 2010.

Gong Zhang is an adjunct professor of applied computer science of University of Winnipeg and research director of Seven Oaks General Hospital Wellness Institute. He is also a research scientist in Rizhao Hospital of Traditional Chinese Medicine, China. His research is focused on big medical data analysis and development of biosensors of point of care system. In 2008 and 2009, he received the top awards for pulse wave analysis for cardiovascular health and H1N1 biosensor from the biomedical commercialization of Canada. He is currently working on developing new wearable system for chronic disease monitoring.